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PERMUTATION STATISTICS, PATTERNS AND MOMENT SEQUENCES

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Which combinatorial sequences correspond to moments of probability measures on the real line? We present two large classes of such sequences, where for one of the classes we prove these to be moment sequences, and conjecture it for the other class:

- We present a generating function CF, as a continued fraction, for a 14-parameter family of integer sequences and interpret these in terms of statistics on permutations and other combinatorial objects. Special cases include several classical and noncommutative probability laws, and a substantial subset of the orthogonalizing measures in the q-Askey scheme of orthogonal polynomials.
  Under mild conditions on the parameters, the sequences arising from CF are moment sequences, and this continued fraction captures a variety of combinatorial sequences, counting various kinds of permutations, set partitions and perfect matchings. In particular, it characterizes the moment sequences associated to the numbers of permutations avoiding classical, vincular and consecutive patterns of length 3.
- Generalizing the notion of descent set of a permutation, we study the number of permutations with a given, arbitrary consecutive pattern occurring at fixed positions. When these positions are at regular intervals we get an enumerating sequence for the permutations in question. We outline a recursive formula for all such 'regular' sequences, and conjecture that they are moment sequences.

This is joint work with Natasha Blitvić and with Blitvić and Slim Kammoun, respectively.