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Interview with Don Knuth

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Photo by Vivian Cromwell

Donald E. Knuth completed his undergraduate studies at Case Institute of Technology in 1960. He obtained a Ph.D. at California Institute of Technology in 1963, under the direction of Marshall Hall, Jr. He joined Stanford University as Professor of Computer Science in 1968. In 1993 he became Professor Emeritus of The Art of Computer Programming, at Stanford University. Professor Knuth has held visiting positions at the University of Oslo (1972–1973) and the University of Oxford (2002–2017). He has spoken at many conferences, including an invited talk at the International Congress of Mathematicians in Nice in 1970; the American Mathematical Society's Gibbs Lecture in 1978; the SIAM von Neumann Lecture in 2016. Professor Knuth has received several awards, including the ACM A.M. Turing Award in 1974, the Medal of Science from President Carter in 1979, the American Mathematical Society's Steele Prize for expository writing in 1986, the Benjamin Franklin Medal in 1988, the Adelsköld Medal from the Swedish Academy of Sciences in 1994, the Harvey Prize from the Technion in 1995, the IEEE's John von Neumann Medal in 1995, the Kyoto Prize for Advanced Technology in 1996, the 2010 Frontiers of Knowledge Award in Information and Communication Technologies, and the IET's Michael Faraday Medal in 2011. Knuth was elected to the National Academy of Sciences in 1975, the National Academy of Engineering in 1981, and the American Philosophical Society in 2012. He is a foreign associate of the French Academy of Sciences, the Norwegian Academy of Science and Letters, the Bavarian Academy of Sciences, the Russian Academy of Sciences, and a Foreign Member of the Royal Society of London. He is a Fellow of the American Academy of Arts and Sciences, the British Computer Society, the Society for Industrial and Applied Mathematics, the American Mathematical Society, the Association for Computing Machinery, and the Computer History Museum; also an Honorary Member of the Institute for Electrical and Electronics Engineers and the London Mathematical Society. Professor Knuth holds honorary doctorates from more than 30 universities around the world, including Harvard University, Oxford University, the University of Paris, the Royal Institute of Technology in Stockholm, the University of St. Petersburg, the Université de Montréal, the University of Tübigen, the University of Oslo, the Swiss Federal Institute of Technology in Zürich, the University of Bordeaux, St. Andrews University, the National University of Ireland, the University of Antwerp in Belgium, Masaryk University in Czechia, and the University of Macedonia in Greece. Together with Herbert Wilf, Professor Knuth cofounded the Journal of Algorithms in 1979, and he has served as a member of the editorial board in numerous journals, including Advances in Mathematics (1971–1979), Combinatorica (1985–1998), Discrete and Computational Geometry (1986–2012), Discrete Mathematics (1970–1978), Electronic Journal of Combinatorics (1994–2013), Historia Mathematica (1972–1979), Journal of Graph Theory (1975–1979),

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Journal of the ACM (1964–1967), Random Structures & Algorithms (1990–2007), SIAM Journal on Computing (1973–1979). The minor planet “(21656) Knuth” was named for him in 2001.

Mansour: Professor Knuth, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Knuth: Aha, it is already clear that you are going to be asking some great questions! I guess I can answer best by oversimplifying, since such questions can never be fully answered.

Many years ago I tried to answer the question “What is mathematics?” by saying that mathematics is what mathematicians do. I also said that the answer to “What is computer science?” is analogous[1].

More precisely, I believe that different people have different ways of organizing knowledge in their heads and that fields of study are principally defined by the kinds of reasoning that are most in tune with the way their practitioners think. (For example, I'm a computer scientist because my brain resonates with many aspects of computation.) I studied random samples of the literature of mathematics and of computer science, trying to imagine what sorts of knowledge and paradigms I would have to teach a computer, in order for it to have developed the ideas expressed there.

Some people think computer science is a subset of mathematics; others think mathematics is a subset of computer science. I believe neither is true, but that there is a large intersection. The union of the two is the kind of knowledge that is created by human beings, rather than present in the natural universe. Unlike physicists, chemists, and biologists, we get to make up our own ground rules. Computer science and mathematics have, in turn, many subfields, and I associate them with different kinds of thinking. Vive la différence — such diversity is wonderful.

I learned my favorite definition of mathematics from Andy Gleason, “mathematics is the science of patterns.” And I learned my favorite definition of combinatorics from my advisor Marshall Hall. Here is how I formulate it on page 1 of my book *Combinatorial Algorithms*[2]: “Combinatorics is the study of the ways in which discrete objects can be arranged into various kinds of patterns. ... Five basic types of questions typically arise when combinatorial problems are studied, in increasing order of difficulty: (i) Existence: Are there any arrangements X that conform to the pattern? (ii) Construction: If so, can such an X be found quickly? (iii) Enumeration: How many different arrangements X exist? (iv) Generation: Can all arrangements X_1, X_2, ... be visited systematically? (v) Optimization: What arrangements maximize or minimize $f(X)$, given an objective function f?”

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Knuth: There clearly are tremendously helpful interactions in both directions. For example, algebraic patterns can often be translated into equivalent geometric patterns, and vice versa, allowing different kinds of intuition with which we can gain insights. Analytic functions of complex variables allow us to enumerate complicated combinatorial patterns; conversely, such patterns point to yet-undeveloped aspects of complex analysis. Random variables demonstrate the existence of patterns that we don't know to construct. And so on.

Mansour: What have been some of the main goals of your research?

Knuth: I'm primarily a teacher and writer who tries to organize existing knowledge and pass it on to new generations of enthusiasts. While juxtaposing such ideas I naturally run across a multitude of questions that have not been fully explored, so I try to see how they fit into the story that I'm trying to tell. I also try my best to understand and preserve the history of those ideas.

Mansour: We would like to ask you about your formative years. What were your early

[1] D.E. Knuth, *Computer science and its relation to mathematics*, Amer. Math. Monthly 81 (1974) 323–343.

[2] D.E. Knuth, *The art of computer programming*, Vol. 4A, *Combinatorial algorithms*, Part 1. Addison-Wesley, Upper Saddle River, NJ, 2011.

experiences with mathematics? Did that happen under the influence of your family or some other people?

Knuth: My teachers before college were not very good at mathematics. But I liked to play with the elementary ideas that I was exposed to; for example, I spent hundreds of hours drawing graphs of polynomial functions, with just one of the coefficients varying. I also played little games, with horses moving around a track, by throwing dice to determine how far each horse would move. I mostly sat in my bedroom doing this. My father was a teacher who worked part-time as an accountant; he had a machine that could multiply numbers together and print the answers on a little paper tape.

Mansour: Were there specific problems that made you first interested in combinatorics?

Knuth: In my senior year I got very interested in orthogonal latin squares, because of lectures by visiting professor R.C. Bose. It was 1959, the year that Bose and others disproved Euler's famous conjecture about nonexistence[3,4,5].

Mansour: What was the reason you chose the California Institute of Technology for your Ph.D. and your advisor Marshall Hall, Jr.?

Knuth: Bose recommended it highly. Among my other teachers at Caltech were Robert Dilworth, Herbert Ryser, and Adriano Garsia (although Garsia worked at that time in Fourier analysis). Dick de Bruijn was also a frequent visitor.

Mansour: What was the problem you worked on in your thesis?

Knuth: I found an infinite family of non-Desarguesian projective planes of orders 32, 64, 128, etc., and found further structural properties of finite planes[6,7].

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Knuth: My research has always been a consequence of curiosity. I guess I learned early on how to ask questions whose answers might be instructive. Sometimes the questions are to develop a theory—for example, to find the consequences of some given axioms, or to invent appropriate axioms. Sometimes the questions are to explain a pattern of numbers. Often the questions are to figure out how fast a particular algorithm runs when presented with a particular kind of input. Sometimes the questions are to understand a historical development.

I avoid questions about things for which I do not have a decent intuition. For example, I'm not good at visualizing objects in more than two dimensions.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Knuth: I typically examine many small cases of related problems, in order to learn not only what is true but what is false.

When I'm trying to prove that something is true, I find I can usually do better if I try to find a counter-example—because I'm a pretty good nit-picker. Then, when I realize why I'm stuck and can not find a counter-example, I might, in fact see a proof.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Knuth: I generally do not like questions about the "most influential" or "most important" results, because I think combinatorics (like any significant subject) advances mostly by thousands of small steps rather than by a few big ones. Combinatorics is analogous to the great wall of China, consisting of many, many bricks marvelously put together.

However, since you asked, I guess the most influential results since 1990 that come to mind are (i) the breakthrough in solution methods for satisfiability problems, based on "conflict-driven clause learning"[8,9]; (ii) automated proofs of identities, as in the book

[3] R.C. Bose and S.S. Shrikhande, *On the falsity of Euler's conjecture about the non-existence of two orthogonal latin squares of order* $4t+2$, Proc. Nat. Acad. Sci. U.S.A. 45 (1959) 734–737.

[4] R.C. Bose and S.S. Shrikhande, *On the construction of sets of mutually orthogonal latin squares and the falsity of a conjecture of Euler*, Trans. Amer. Math. Soc. 95 (1960) 191–209.

[5] R.C. Bose, S.S. Shrikhande, and E.T. Parker, *Further results on the construction of mutually orthogonal latin squares and the falsity of Euler's conjecture*, Canad. J. Math. 12 (1960) 189–203.

[6] D.E. Knuth, *Finite semifields and projective planes*, J. Algebra 2 (1965) 182–217.

[7] D.E. Knuth, *A class of projective planes*, Trans. Amer. Math. Soc. 115 (1965) 541–549.

[8] J.P. Marques-Silva and K.A. Sakallah, *GRASP: A Search Algorithm for Propositional Satisfiability*, IEEE Transactions on Computers 48:5 (1999) 506–521.

[9] M.W. Moskewicz, C.F. Madigan, Y. Zhao, L. Zhang, and S. Malik, *Chaff: Engineering an efficient SAT solver*, ACM/IEEE Design Automation Conf. 38 (2001) 530–535.

"$A = B$" by Petkovšek, Wilf, and Zeilberger[10]; (iii) the sequence of 23 papers by Robertson and Seymour (1983–2012) about the theory of graph minors[11,12].

Mansour: What are the top three open questions in your list?

Knuth: Again I'm not happy with a "top three" question, but I don't want to duck it. Of course, there are lots and lots of unsolved problems for which I would dearly love to know the answer, but the best of them are problems for which partial results are also likely to be fruitful. I guess I vote for (i) to prove the exponential time hypothesis[13] (ETH), namely that $\inf\{\log_2 \tau \mid$ we can know an algorithm to solve in τ^n steps the satisfiability problem with clauses of size 3 in n variables$\} > 0$; (ii) a nonconstructive proof that P = NP, by showing that only finitely essentially different cases can arise, although we may never know when we have found them all (something like Robertson and Seymour did for graph minors); (iii) a study of the curious sign pattern that arises in the asymptotics of the Gould numbers—the number of set partitions whose tail is a singleton—as illustrated in the answer to exercise 7.2.2.1–190 of *The Art of Computer Programming* **4** (TAOCP), fascicle 5 (2019), pages 146 and 277. The latter problem is of course presumably much easier than (i) or (ii); yet I do believe it is a good representative of the many currently unsolved problems whose solution will probably have nice spinoffs.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

Knuth: Let me put in a plug for problems 1, 2, 3, and 5 that were highlighted in my "Flajolet lecture" of 2014[14].

I'm especially interested in problem 5, which is about the "principle of negligible perturbation." I have always had high hopes for the analyses of important combinatorial algorithms that would make use of this simple-yet-subtle principle, which was illustrated in the paper I wrote with Rajeev Motwani and Boris Pittel on page 1 of volume 1 of *Random Structures & Algorithms*[15], then again in a later paper with Svante Janson[16]. But so far I'm surprised and a bit discouraged that nobody else has yet picked up on the idea.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Knuth: I do not like to say that any topic is more important than another (although I expect that I will *never* be enthused about problems that have been posed about Smarandache numbers). According to the philosophy I presented earlier, the importance of a problem is relative to each researcher's experience, intuition, and personal way of structuring knowledge. One should not work on a problem just because somebody else tells you it is important or will make you rich and famous. Work on a problem if you think it has your name on it because of your particular skills, and if you think the solution will be relevant to others.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Knuth: See my lectures entitled "Theory and practice" in the book *Selected Papers on Computer Science*[17] (1996), pages 123–127, 129–139, 141–147, 149–167; see also "Theory and practice and fun" in *Companion to the Papers of Donald Knuth*[18] (2012), pages 39–40.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Knuth: Ask if they are sure they would not prefer a research career in computer science (based of course on their own profile of abilities). And to recommend that they not be carried away by trendy stuff; they should trust their own ideas of beauty.

Omer Reingold asked me a similar question

[10] M. Petkovšek, H.S. Wilf, and D. Zeilberger, $A = B$, with a foreword by D.E. Knuth, with a separately available computer disk, A K Peters, Ltd., Wellesley, MA, 1996.

[11] N. Robertson and P.D. Seymour, *Graph minors. I. Excluding a forest*, J. Combin. Theory Ser. B 35:1 (1983) 39–61.

[12] N. Robertson and P.D. Seymour, Graph minors. XX. Wagner's conjecture, J. Combin. Theory Ser. B 92:2 (2004) 325–357.

[13] R. Impagliazzo and R. Paturi, *On the complexity of k-SAT*, J. Comput. Syst. Sci. 62:2 (2001) 367–375.

[14] `https://www-cs-faculty.stanford.edu/~knuth/flaj2014.pdf`

[15] D.E. Knuth, R. Motwani, and B.G. Pittel, *Stable husbands*, Random Struct. Algorithms 1:1 (1990) 1–14.

[16] S. Janson and D.E. Knuth, *Shellsort with three increments*, Random Struct. Algorithms 10:1–2 (1997) 125–142.

[17] `https://www-cs-faculty.stanford.edu/~knuth/cl.html`

[18] `https://www-cs-faculty.stanford.edu/~knuth/cp.html`

three years ago, and he posted my answers on the Stanford CS theory research blog[19,20,21].

Mansour: You are a prolific researcher, writer and programmer. Do you have some daily routines and rituals that optimize your productivity? What advice would you give on this topic?

Knuth: Some tricks that work for me, besides not having a TV set or a cell phone: (i) When I read a paper I try always to guess what is coming, before turning the page. I try to prove a theorem before looking at the proof. (Usually I fail, of course; but I'm more ready to appreciate the solution, and meanwhile I have learned another technique.) I try to change the notation, as a help for my own thinking, and to translate things into equivalent forms. (ii) I write my first drafts in pencil, because I can type faster than I can think. I have a very comfortable chair in which I can sit when doing this. But when I feed that draft to a computer, I stand up at the terminal and polish the material as I type. (Martin Gardner introduced me to the virtues of a stand-up desk in 1972.) (iii) When feasible, I write a program to get familiar with the ideas. "Literate programming"[22] is a big win here. (iv) I take a daily siesta. I go to sleep when tired, and do not use stimulants to keep awake and alert. (v) Inspired by Rex Stout's detective Nero Wolfe, I schedule my interactions with others, instead of being randomly interruptible. (vi) When possible I try to be working on only one thing at a time. (Computer scientists call this "batch processing." The opposite is "swap-in-swap-out.") (vii) When I have to decide what to do next, I choose to work on the thing that I enjoy the *least*, unless I have a really good reason to procrastinate. For if some non-fun thing has got to be done eventually, and if there will be no better time than the present, it is best to grin and bear it.

Mansour: Would you tell us about your interests besides mathematics?

Knuth: I see that you are about to ask me about music and books. So I guess that leaves exercise, movies, religion, and food? (i) I try to swim a few laps, four times a week. (ii) I love classic films (e.g., think of Harold Lloyd, Buster Keaton, Humphrey Bogart, Audrey Hepburn; Frank Capra, Alfred Hitchcock, Akira Kurosawa, David Lean, Satyajit Ray; musical comedies (Oklahoma, The Band Wagon, West Side Story, Camelot, ...); Attenborough's Gandhi; Monty Python; animation (Fantasia, Yellow Submarine, Ratatouille, Up, WALL-E, Coco, ...). (iii) I gave six lectures at the Massachusetts Institute of Technology (MIT) on connections between science and religion—on the value of mysteries that give us humility, to complement the certainties that give us structure (see my book *Things a Computer Scientist Rarely Talks About*[23]). (iv) When I cook for myself, I often favor the "Venn diagram method": No stirring, so that I can taste each subset of the ingredients. (It is sort of the opposite of Persi Diaconis's "mixing-time method.")

Mansour: We read from your personal web page that you are also interested in composing music. How would you compare these two creative processes: proving theorems and composing musical pieces? Which one do you think is harder: writing a math paper that can be published at Annals of Mathematics or composing a symphony that can be compared to those of Mozart or Beethoven?

Knuth: I believe the creative part is amazingly similar, also if you would have asked me about writing a poem or a computer program or making a sculpture, etc. The aim is not to get recognition or prestige. It is rather to communicate a personal vision to other people, hoping that others will understand why you think that your music or your math or your poem, etc., adds to the fruits of civilization.

Mansour: You play piano and organ. Do you perform only at home with your family and friends? If you are asked to create a playlist specifically for combinatorialists, which pieces would your playlist include? Of course, we will add your *Fantasia Apocalyptica*[24] to the playlist!

Knuth: I'm not good enough to perform in public; the only exception was an organ duet

[19] `https://theorydish.blog/2018/02/01/donald-knuth-on-doing-research`
[20] `https://theorydish.blog/2018/02/26/donald-knuth-on-writing-up-research`
[21] `https://theorydish.blog/2018/06/04/don-knuth-on-general-principles`
[22] `https://www-cs-faculty.stanford.edu/~knuth/lp.html`
[23] `https://www-cs-faculty.stanford.edu/~knuth/things.html`
[24] `https://www-cs-faculty.stanford.edu/~knuth/fant.html`

concert given once in Waterloo Ontario. I do occasionally accompany a choir or string quartet. Thank you for mentioning my organ composition, on which I spent many years! Your playlist should definitely include pieces by Noam Elkies[25] and Peter Winkler[26]. I believe Bach, Tchaikovsky, Brahms, Gershwin, Stravinsky, Hindemith, Bernstein were combinatorialists at heart; this is evident from the scores of their compositions. (Gershwin, in particular, took lessons in combinatorics from the eccentric teacher Joseph Schillinger, and I think he put them to good use.)

My litmus test for the quality of a composition is whether, after hearing it six times, you recognize it and enjoy it on the seventh. Of course, not all compositions by a great composer will pass this test; nor will the same composition with different listeners.

Mansour: It seems that you are also an avid book reader. Which books are you reading these days? Have ever thought about writing a novel? If yes, what would it be about?

Knuth: I read "literature" mostly to help me go to sleep, not for edification. So I know dozens and dozens of works by Erle Stanley Gardner, Rex Stout, Agatha Christie, Dorothy Sayers, P. D. James, Robert Bernard, Robert B. Parker, Raymond Chandler, Frederick Forsyth, Maj Sjöwall and Per Wahlöö, Ken Follett, Ian Fleming, Sara Paretsky, Herman Wouk. I like Tolstoy, but dislike Dostoyevsky. Hated Durrell's Alexandria Quartet (too sloppy). I mention my favorites on the webpage `https://www-cs-faculty.stanford.edu/~knuth/retd.html`.

But OK, since you asked, let me also tell you what I have read most recently: *Oliver Twist* by Dickens; *Uncle Tom's Cabin* by Stowe; *Death Times Three* by Stout; *The Regatta Mystery* by Christie; *Without Feathers* by Allen. Just now I'm re-reading *Days of a Man*, Volumes 1 and 2, by David Starr Jordan (Stanford's first president), together with the recent *Why Fish Don't Exist* by Lulu Miller (which exposes his dark side). I'm also reading a wonderfully documented biography of Stravinsky, by Vera Stravinsky and Robert Craft; but that one is too interesting—it does not help me sleep—so I have to sample it carefully.

You also ask about writing a novel? I don't think I have the talent. (I did write a mathematical novelette, *Surreal Numbers*[27], and the experience was immensely enjoyable. But that little paperback certainly does not deserve to be classed as a novel. Think of an opera, which consists of good music with a little bit of a plot; *Surreal Numbers* is good math, with a little bit of a plot.)

I have toyed with the idea of writing twin short stories, entitled "The Window": In one of them, a window stays shut; in the other, the protagonist begins by opening it. Everything else in both stories starts from the same initial state of the universe. But their endings are completely different. (This idea was inspired by a remark near the end of Mark Twain's book *The Mysterious Stranger*. I will almost certainly never get around to fleshing it out, because I have too many other things to do. And anyway, Tom Tykwer's movie *Run, Lola, Run* was based on a similar idea and he exploited it much better than I ever could.)

I have also wondered if it would be possible to write an engaging short story or even a novel whose protagonist is an ant colony. Individual ants are like the cells of a human body, but the colony as a whole has consciousness.

Let somebody else follow up on such things! I'm better suited to writing about programming.

Mansour: You frequently propose problems to some math magazines, especially to Monthly. Where do these problems come from? Do you dedicate some special time to create such interesting questions?

Knuth: The problems arise naturally as I'm writing new material for *The Art of Computer Programming*, because I'm always asking myself questions. When the answer looks particularly instructive, and not strongly enough related to programming to be a suitable exercise in my book, I submit it as a problem.

Mansour: You have created numerous computer programs. How do you usually decide to write a computer program? Was TEX invented or discovered? What motivated you for it?

Knuth: I write roughly five programs every week, to try out and test ideas before putting them into *TAOCP*. Some of the programs are

[25] `https://people.math.harvard.edu/~elkies/music.html`
[26] `https://math.dartmouth.edu/~pw/music/rags.html`
[27] `https://www-cs-faculty.stanford.edu/~knuth/sn.html`

very short; others are rather extensive; but I invariably find that I understand something better after I have tried to explain that thing to a computer.

The story of TEX's birth has been told many times. For a short version, see pages 1–12 of my book *Digital Typography*[28]. But my favorite version appeared as part of a paper called "The errors of TEX," which has been reprinted on pages 243–291 of *Literate Programming*[21], especially pages 249–266.

Mansour: Your life-long book project *The Art of Computer Programming* is a great inheritance to the scientific community. How did you decide on such an enormous project? What do you think about how recent developments in machine learning and quantum computing affect your project?

Knuth: Thank you. TAOCP started in 1962, when I was a second-year graduate student. A representative of the Addison–Wesley publishing house took me to lunch and encouraged me to write a book about software, because one of his editors had suggested that I might be able to do a decent job. This prospect thrilled me, because Addison–Wesley had published my favorite undergraduate texts.

I did not know that it was difficult to write a book. Nor did I foresee how much needed to be clarified, or how much or how soon computer science would grow. So I told them I would be happy to start, as soon as I had completed my Ph.D. thesis.

I froze the table of contents in 1962. Thus "machine learning" and "quantum computing" are entirely orthogonal to the contents of my books, now or in future. And that is good, because those topics involve entirely different paradigms, which I'm no good at.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Knuth: In the old days, enumeration helped me know that I was not missing anything. Nowadays, enumeration is wonderful because I merely need to compute the first few values that relate to whatever problem is currently puzzling me; then the OEIS[29] will tell me what papers to look at.

Mansour: "*Journal of Algorithms* is a great title. Surely there must be a journal of that name someday" — this is a fragment from your letter to editors of the above-mentioned journal. Do you have any comments on our newly launched journal *Enumerative Combinatorics and Applications*?

Knuth: I'm especially happy to see your combined emphasis on first-rate quality and totally open access.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Knuth: I guess I'm most proud of my work on "The birth of the giant component," with Svante Janson, Tomasz Łuczak, and Boris Pittel, because it once was the subject of an entire issue of *Random Structures & Algorithms*[30]. (Reprinted with corrections and an addendum on pages 643–792 of my *Selected Papers on Discrete Mathematics*[31] (2003).)

The problem started with a rumor. We heard at Stanford that students of Dick Karp at Berkeley had simulated evolution of random graphs with the Erdős–Rényi model and discovered that, with high probability, at most one component was "complex" (not a tree) at any time throughout the entire evolution. Therefore Boris and I begin to study the steps when cycles first appeared; for if the rumor were true, it seemed likely that even more surprises would be just around the corner.

But the rumor was misleading: The true probability that at most one complex component is present, throughout the evolution, turns out to be asymptotically $5\pi/18 \approx 87\%$(!). Additional complex components do tend to be present in the other 13% of the cases; but only for a very brief time. That is why the Berkeley students did not see it in their samples.

If we had known how intricate the analysis would turn out to be, I doubt if we would have

[28] https://www-cs-faculty.stanford.edu/~knuth/dt.html

[29] https://oeis.org

[30] S. Janson, D.E. Knuth, T. Łuczak, and B. Pittel, *The birth of the giant component*, with an introduction by the editors, Random Struct. Algorithms 4:3 (1993) 231–358.

[31] https://www-cs-faculty.stanford.edu/~knuth/dm.html

had the courage to begin. (Just as I would never have begun *The Art of Computer Programming* in the 60s if I would have known how much computer science was destined to grow.) Many of the best breaks I have had in life have therefore been due to being extremely bad at estimating the difficulty of a project.

There was indeed a definite "Aha" moment for me while we were writing that paper. It occurred at about 3am one night, when I was drawing a diagram to summarize some of the recent calculations we had made. We had figured out how to use complex analysis at a double saddle point to calculate the asymptotic probabilities of the most fundamental state transitions that take place when the "big bang at the double pole" is slowed down to the tiniest steps. This diagram is now Fig. 1, on page 301 of the paper (page 722 of the book). It shows the probabilities of the three possible states after three "collisions" have occurred; and those probabilities are rational numbers with denominator 17017.

Aha! 17017 is 17 times 13 times 11 times 7! That can not be a coincidence—there must be a reason! And the denominator after four "collisions" was 7436429. Those factorable numbers told me where to look, in order to simplify an exponential generating function and to decipher the whole structure.

I cannot resist telling more of that story. On the next morning, it turned out that Bill Gates was visiting Stanford, because he was being wooed by our fundraisers in hopes of getting money for a new Computer Science building. Although I'd had little sleep, I was asked to tell him what I had been working on; so I drew the diagram on a blackboard, and explained how 17017 was the key to success. Later that day, he agreed to donate millions of dollars, and he told the fundraising team that he had been "especially impressed by Don Knuth's enthusiasm for research."

The fundraisers asked me to recreate that blackboard display, so that they could take an archival photograph. Consequently nobody can claim that theoretical computer science isn't practical.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Knuth: *The Art of Computer Programming*. I have published 3772 pages so far, and have drafted 116 further pages. I hope to draft another page tomorrow. The current drafts are online[32], so that readers can help me remove errors before publication.

Mansour: My last question is philosophical. Have you figured out why we are here? If yes, would you tell us the answer in an encrypted way so that only those, among our readers, who put some effort can learn it?

Knuth: Well, I mentioned mystery and humility earlier. My best shot at an answer to your "last question" (whew!) appears on pages 17 and 149 of that MIT-related book.

In conclusion, let me thank you for posing such an array of interesting questions. Oh, how I wish somebody had asked my favorite mathematicians of the past to fill out such a questionnaire! What would Euler have said? And Bourbaki?!

Mansour: Professor Donald Knuth, I would like to thank you for this very interesting interview on behalf of the journal *Enumerative Combinatorics and Applications*.

[32] `https://www-cs-faculty.stanford.edu/~knuth/news.html`