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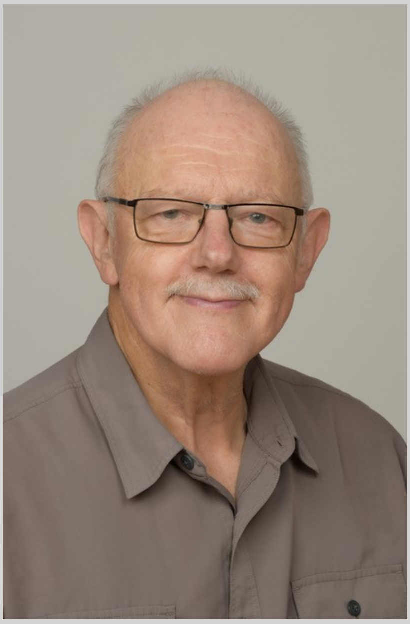
Interview with Tomaž Pisanski

Toufik Mansour



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Tomaž Pisanski studied at the University of Ljubljana where he obtained a B.Sc., M.Sc., and Ph.D. in mathematics. He completed his Ph.D. thesis under the guidance of Torrence Parsons in 1981. He also obtained an M.Sc. in computer science from Pennsylvania State University in 1979. He worked at the University of Ljubljana as an Assistant (1972 – 1982), Assistant Professor (1982 – 1987), Associate Professor (1987 – 1993), and Professor (1993 – 2017). Currently, Pisanski is a professor of discrete and computational mathematics and head of the Department of Information Sciences and Technology at the University of Primorska in Koper. Professor Pisanski has given numerous invited talks in conferences and seminars. He held visiting positions at various research institutes including the University of Zagreb, University of Udine, University of Leoben, California State University, Chico, Simon Fraser University, University of Auckland, and Colgate University. Pisanski has received several awards, including *Prometheus of science for excellence in communication*, for life achievements (Slovenian Science Foundation, 2020), the Donald Michie and Alan Turing Prize for lifetime achievements in Information Science in Slovenia (2016), the Zois Award (the highest national awards for outstanding achievements of Slovenian scientists) (2015), Membership in Academia Europaea (2012), Fellow of the Institute of Combinatorics and Its Applications (2010), and Order of Merit of the Republic of Slovenia (2005). Professor Pisanski is a founding editor of Ars Mathematica Contemporanea (2008) and The Art of Discrete and Applied Mathematics (2018).

Mansour: Professor Pisanski, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Pisanski: Thanks for inviting me.

Mathematicians use some terms, such as number, integral, space, geometry, combinatorics that are elusive, with no precise meaning. Moreover, their vague meanings evolve as mathematics grows and develops. For instance, geometry today is not the same as geometry in ancient Greece. Therefore, I prefer to view combinatorics not as part of mathematics but as an *aspect* of mathematics, perceiving it from the discrete viewpoint. Let me give you just an example supporting my view. Usually, groups have too much structure to be considered as combinatorial objects. However, by selecting a suitable set of generators, one turns the group into a Cayley graph, a typical object of algebraic graph theory and suddenly a whole arsenal of combinatorial tools becomes available to investigate groups. This is an approach where combinatorics easily goes beyond finite or countable.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

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Pisanski: There are two great paradigms, the discrete and the continuous, that have driven the rational human mind since ancient history. Recall, for instance, presocratic philosophers Zeno, Democritus, Pythagoras. Their thought is deeply rooted in the discrete paradigm. The continuous paradigm got an upper hand ever since Newton and Leibniz introduced calculus that evolved in mainstream mathematics and physics. With the advent of computers and digitalization of society, discrete paradigm underlying discrete mathematics turned out to be equally important. Unfortunately, the majority of mainstream mathematicians refuse to accept this fact. There are notable exceptions, such as Carl F. Gauss, who held discrete paradigm in high esteem. Recall his saying that number theory is the queen of mathematics. If he lived today, I would not be surprised if he exchanged the term *number theory* by *combinatorics*.

Mansour: What have been some of the main goals of your research?

Pisanski: Well, whenever I encountered a topic that I found interesting, I first tried to understood it. I was sure I understand it, when I was able to explain it to others through examples, counter-examples, and above all when I could solve problems. Only when I began my graduate studies, I realised that I really understand a topic if I can explain it to the computer. In 1976, I wrote a computer program in Pascal for computing the homology groups of abstract simplicial complexes as a project in my graduate seminar. By programming Smith's normal form from scratch I learned well the structure of finitely generated abelian groups. I was able to use this knowledge later when I contributed to computing the genus of some families of finite abelian and hamiltonian groups. Moreover, this approach made me a fan of the discrete paradigm. I did learn some real and complex analysis, functional analysis, C*-algebras, topological groups, algebraic topology, etc. However, I never acquired a working knowledge of epsilons and deltas.

Sometimes I used generalization and analogy to develop new ideas. For instance, I was fascinated to see that the 2-manifolds that we learned via maps and atlases can be described purely combinatorially. Each closed surface can be defined as an equivalence class of maps. Each map admits a simple combinatorial description that Gerhard Ringel in his book *Map Color Theorem*[1] calls a *scheme*. Two maps are equivalent if they are embedded on the same surface. The equivalence of maps can be checked by a simple algorithm that transforms each scheme to the one describing the standard fundamental polygon of the corresponding surface. When I found out that the solution of the Heawood map color theorem can be best described by covering graphs, discrete analogs of covering spaces, I proposed to my senior colleague to use a similar analogy to make a combinatorial theory of fibre bundles. We laid down the theory of graph bundles in an 80-page manuscript that unfortunately remained in a preprint form.

Other sources of problems that I like come from scientists, such as theoretical chemists, physicists, and synthetic biologists. In the eighties, I learned about the transfer method when computing matching polynomials of certain families of molecular graphs. Namely, in chemistry, double bonds can be modelled as matchings and Kekulé structures as perfect matchings. Together with my former student Bojan Mohar[2], we computed the notorious Wiener index of trees in linear time. With Croatian chemical physicist Ante Graovac[3], we studied the Wiener index for graphs with symmetries and introduced a modification of this concept that is nowadays called the Graovac-Pisanski index of a graph. About half of my work has been published in non-mathematical journals.

At some point, I became interested in graph drawings, more precisely with graph representations. Motivation again comes from chemistry, where certain molecular graphs such as buckminster fullerene or truncated icosahedron "know" how to position themselves in space. I still do not have a satisfactory answer to this question that nature has a clear answer to. Unit distance graphs represent a particular geometric representation of certain graphs. For instance, we have shown that all generalized Petersen graphs admit such a representa-

[1] G. Ringel, *Map Color Theorem*, Springer-Verlag Berlin Heidelberg, 1974.
[2] B. Mohar and T. Pisanski, *How to compute the Wiener index of a graph* J. Math. Chem. 2 (3), (1988) 267–277.
[3] A. Graovac and T. Pisanski, *On the Wiener index of a graph*, J. Math. Chem. 8 (1991), 53–62.

tion[4]. I became interested in the polyhedral representation of maps on surfaces[5] and later in geometric representations of combinatorial configurations[6,7,8]. Usually, the practical approach via computer experimentation eventually leads to theoretical results.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Pisanski: It all started really bad. In the second grade, I was ranked next to last in the classroom. The low point was the fifth grade where I ended the first semester with an F in mathematics.

And then, something started to happen. During the semester break after my fiasco with mathematics, my father bought me a special notebook. Every morning he would assign five problems from the textbook. After returning from work he would check the solutions and depending on my success he would let me play with friends or assign extra problems from the same topic that I had to solve the same day. The cure slowly started to work. In the seventh grade, we got a new math teacher, a friend of my grandfather's. She let me tutor my schoolmates who had difficulties with math. She only helped me choose the problems that we would later solve in front of a blackboard. Suddenly my self-esteem started to grow. I also learned the fact that the only way I could tell if I understand something is to explain it to others. At home, the wealthy parents of my friend who lived nearby asked me to tutor him, and I got paid. By the end of the year, I was able to buy myself a pair of skates from my savings. My future was secure. I had learned the skill to earn money by using my brains the way I enjoy. Three years later, I was tutoring someone who was two years ahead of me. Since I had to switch schools when entering high school, this gave me a chance for a fresh start. I decided to follow the lectures. During math classes, I would be solving all problems from the textbook, in advance.

At that time I firmly believed there had been nothing new in mathematics since Euclid and Pythagoras. As a mechanical engineer, like my father, I could design new machines, as a chemist I could discover new molecules. A computer programmer? No way! We were afraid of computers. There were rumors that programming computers are a very intensive and dangerous job. Even the best minds cannot do it for a long time, and they are at high risk of going mad since thinking in terms of 0s and 1s is so abstract and unnatural.

However, after a month of high school (9th grade), the picture of math that I had changed completely. Out of curiosity, I attended an evening course for high-school students offered by the Slovenian Math Society on selected mathematical topics. After the first lecture, I understood that everything I had believed about math was wrong. The lecturer was a university professor, and the topic was set theory somehow combined with the history of mathematics. I was shocked when I learned that mathematics is alive, that there are more real numbers than rational numbers, that Galois wrote about math the night before his fatal duel, that Bourbaki wanted to rewrite mathematics, and so on.

After the last lecture on set theory, the lecturer encouraged all the students present to take part in the competition in mathematics that would take place the following Saturday. Since the competition took place at my school, I decided to attend. I was the only competitor to get a perfect score. I was hooked. I competed four times in Slovenian competitions, three times at the federal, Yugoslav, level and twice at the International Mathematical Olympiad (IMO).

Mansour: Were there specific problems that made you first interested in combinatorics?

Pisanski: My interests were always somewhere between discrete mathematics and the-

[4] A. Žitnik, B. Horvat, and T. Pisanski, *All generalized Petersen graphs are unit-distance graphs*, J. Korean Math. Soc. 49:3 (2012), 475–491.

[5] T. Pisanski and M. Randić, *Bridges between geometry and graph theory. Geometry at work*, 174–194, MAA Notes 53, Math. Assoc. America, Washington, DC, 2000.

[6] T. Pisanski and A. Žitnik, *Representing graphs and maps. Topics in topological graph theory*, 151–180, Encyclopedia Math. Appl. 128, Cambridge Univ. Press, Cambridge, 2009.

[7] T. Pisanski and B. Servatius, *Configurations from a graphical viewpoint*, Birkhäuser Advanced Texts: Basler Lehrbücher, Birkhäuser/Springer, New York, 2013.

[8] J. Bokowski, J. Kovič, T. Pisanski, and A. Žitnik, *Combinatorial configurations, quasiline arrangements, and systems of curves on surfaces*, Ars Math. Contemp. 14:1 (2018), 97–116.

oretical computer science. The third year of my undergraduate studies 1969/70 I spent in Nancy, France, as an exchange student. There I took three courses in the Mathematics Department and one course (Formal languages and automata theory) in Computer Science Department with Claude Pair. He also gave me his handouts for a course in graph theory. This was my first formal contact with discrete mathematics.

I tried to generalize regular languages from discrete to continuous by considering words having intervals as their support. When I returned, I prepared together with two of my colleagues (we were all undergraduates) a presentation on generalizing regular languages to infinite support for a local conference and published it in its proceedings. That was the time when my attitude towards computers changed. At my home university, they let me complete the missing courses on my own and I only had to pass the final exams. For instance, I learned Fortran for the newly acquired IMB 11-30 on my own. My individual project was to do one of the exercises from the book *The Art of Computer Programming* by Donald Knuth[9]. I was assigned the task of printing out an arbitrary crossword puzzle with correctly placed numbers in each field which would carry the first letter of a word.

At that time, I made three important discoveries. Computers can help you get results outside of numerical mathematics. The computer was merciless. Until I found and corrected all my mistakes, it would not yield and give me the answer I wanted. However, the reward at the end was fascinating. One could enter numerous patterns of forbidden fields and one would always get a perfect printout. There was no need for further programming. The computer also upgraded my skill of finding and correcting my own mistakes. I first developed this skill during my mathematical competition period. Everyone makes mistakes. However, finding your own mistakes as soon as possible requires a skill. If you leave bugs undiscovered for too long in your evolving computer program, you are in trouble. A few years later with Pascal and structured programming, everything became much easier.

Before graduating I held several part-time jobs. Usually, they involved helping non-mathematicians formulate their ideas in a correct way. For instance, I helped a group of architects formulate irrational grids in two or three dimensions. I published the findings in some local proceedings in 1972. Several years later, I realized that we had rediscovered the Lindenmeyer systems first described in 1968. Another job I did for a few years, involved helping some electrical engineers who were interested in hardware formulate mathematical models correctly. I presented our work on the detection of faults of fanout-free combinatorial networks at conferences in Bulgaria and Paris, France. I knew I liked discrete mathematics and theoretical computer science. I knew I lacked formal education. At that time there was no course in discrete math taught at the mathematics department of my university or any university in Yugoslavia.

For completing my undergraduate program in mathematics, I had to write a diploma thesis. I chose the title "Random number generators", implemented three algorithms for generators from Knuth's book and most of the statistical tests described in the same chapter. And then a miracle happened. In 1971, Ljubljana hosted the world IFIP congress with over 2500 participants from all over the world. Together with my colleagues, I was asked to assume the role of "Technical Secretary". We were in charge of overhead projectors, spare transparencies, flip charts, lights, microphones, etc. I met Donald Knuth, who was a plenary speaker, in person and was able to ask him questions about random number generators, I gave him a copy of our report on continuous languages. It was wonderful! Among the participants of IFIP, there was also Robert Korfhage, who later visited Ljubljana as part of his sabbatical. He volunteered to offer an introductory fast informal course in graph theory. For the first part, he used Harary's textbook. For the second part, he would pick some recent papers that were available in our library and describe their contents to the audience. He also prepared some handouts. I was definitely hooked. I wanted to be a graph theorist.

However, there were problems. Towards the

[9]D.E. Knuth, *The art of computer programming*, Vol. 4A, Combinatorial algorithms, Part 1. Addison-Wesley, Upper Saddle River, NJ, 2011.

end of my graduate studies, the proposed title of my masters thesis *Algorithms on graphs* was rejected since my advisor-to-be, was not qualified in graph theory. But at that time there was nobody qualified in graph theory! My new advisor gave me the recently published Ringel's book Map Color Theorem[1] and asked me to write my masters thesis on Heawood's Map color theorem[10] together with proofs of all supportive results including the Jordan-Schoenflies Theorem[11]. It took me a few extra years to complete my master's degree. My studies were interrupted by one year of obligatory military service.

I wrote my first paper as a graduate student when I was sick in the military hospital. It was a short note on the existence of certain triangulations that I published in Glasnik Matematički in Zagreb[12]. Later, I learned that these triangulations are duals of fullerenes and that the referee was Branko Grünbaum, whom I first met in person only ten years later in Seattle. A quarter of a century later, we discovered that we share a common interest in configurations of points and lines. Our collaboration started in 2000 at a wonderful conference in Israel, organized by his former student Joseph Zaks. Branko and I wrote four papers[13,14,15,16] together with other mathematicians, including his students and my students.

Mansour: What was the reason you chose Ljubljana University for your Ph.D. and your advisor Torrence Parsons?

Pisanski: There was not much choice. When I started my undergraduate education, Slovenia had only one university that was founded in 1919. From my home, I had a 15 minutes walk to lectures. All the lectures were in my native tongue. Hence the University of Ljubljana was a very convenient, inexpensive choice for me. Soon, after completing my undergraduate studies, I was also employed there.

Another chance brought me to Penn State and Tory Parsons. My sister, who lives in State College, PA, encouraged me to continue my studies there and offered me to stay at their home. I wrote to all the professors who were listed in the Penne State brochure with research on or teaching graph theory. Only Tory replied and wrote to me that there are more courses on graphs in the Computer Science Department than in the Mathematics Department. I got Fulbright and International Research and Exchanges Board Scholarships that enabled me to study in the USA. I had chosen computer science but kept close contact with Tory. When it turned out that I could not have him as an adviser, I cut my studies short, completed the second M.Sc. degree, this time in CS, and returned home to my family. He would come to Ljubljana for my Ph.D. defense. We traveled together to a conference in Hungary, where I met several mathematicians, my future coauthors. Later Tory came to Ljubljana with his family for a sabbatical, and a year later I spent my sabbatical with him in California also with my family. After two semesters in California, I continued my sabbatical at Simon Fraser by the invitation of Brian Alspach. Unfortunately, Tory passed away unexpectedly at the age of 46 during our stay in California.

Mansour: What was the problem you worked on in your thesis?

Pisanski: In my Ph.D. thesis I expanded the original parts of my masters thesis. This was a generalization of White's[17] technique for finding quadrilateral embeddings of cartesian products of certain graphs. I also developed the basics of the theory of branched coverings of graphs that is useful when observing duals

[10] P. J. Heawood, *Map colour theorem*, Quart. J. Pure Appl. Math. 24 (1890), 332–338.
[11] H. Tverberg, *A proof of the Jordan curve theorem*, Bull. London Math. Soc. 12 (1980), 34–38.
[12] T. Pisanski, *On planar graphs with 12 vertices of degree five*, Glasnik Mat. Ser. III 12(32) (1977), no. 2, 233–235.
[13] N. Bašić, J. Grošelj, B. Grünbaum, and T. Pisanski, *Splittable and unsplittable graphs and configurations*, Ars Math. Contemp. 16:1 (2019), 1–17.
[14] M. Boben, B. Grünbaum, and T. Pisanski, *Multilaterals in configurations*, Beitr. Algebra Geom. 54:1 (2013), 263–275.
[15] L. W. Berman, J. Bokowski, B. Grünbaum, and T. Pisanski, *Geometric "floral" configurations*, Canad. Math. Bull. 52:3 (2009), 327–341.
[16] M. Boben, B. Grünbaum, T. Pisanski, and A. Žitnik, *Small triangle-free configurations of points and lines*, Discrete Comput. Geom. 35:3 (2006), 405–427.
[17] A. T. White, *On the genus of the composition of two graphs*, Pacific J. Math. 41 (1972), 275–279.
[18] J. L. Gross, B. Jackson, T. D. Parsons, and T. Pisanski, *Wrapped coverings and their realizations*, Proceedings of the thirteenth Southeastern conference on combinatorics, graph theory and computing (Boca Raton, Fla., 1982), Congr. Numer. 35 (1982), 333–347.
[19] B. Jackson, T. D. Parsons, and T. Pisanski, *A duality theorem for graph embeddings*, J. Graph Theory 5:1 (1981), 55–77.
[20] T. D. Parsons, T. Pisanski, and B. Jackson, *Dual imbeddings and wrapped quasicoverings of graphs*, Discrete Math. 31:1 (1980), 43–52.

of embedded voltage graphs and duals of the corresponding derived graphs[18,19,20].

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Pisanski: I always looked mostly at topics that I found interesting. I just wanted to understand things better. I was mostly interested in generalizations. What happens if you have a theorem and you drop or modify a condition? On the other hand, I soon realized that combinations of different subjects, such as graphs and surfaces, graphs and groups, graphs and geometry, sometimes give most satisfying results.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Pisanski: Yes, I believe, that in abstract mathematics everything is already there, we just have to discover it – and prove it. For me, this is the most plausible explanation of the fact that many results are discovered independently, by unconnected individuals. My intuition tells me something is true. I conjecture and try to prove it. If I am wrong and find a counterexample, I modify the conjecture accordingly and try to prove it, again. In this respect, I believe in the existence of ideas outside our minds. I know that such views have been subject to critique by many wise thinkers. This is certainly only a belief. I have no way of proving it and no intentions of convincing someone who holds opposite views.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Pisanski: My knowledge of mathematics and combinatorics is very one-sided. I mostly follow what was in my research interest. Let me try to answer in general terms.

- Combinatorial species[21].
- Oriented matroids[22].
- Large networks, Barabási-Albert model[23].

Above all, it is computer-assisted mathematics. The number of mathematicians that rely in their research on various computer systems, including symbolic computations, ranging from, say, Haskell, to GAP, magma, SageMath, Mathematica, etc., has increased tremendously in the last thirty years. I have used them in all my recent papers.

Mansour: What are the top three open questions in your list?

Pisanski: I usually do not rank problems. I am interested in several topics. Here I have chosen three of them that I have worked on since I turned 50.

– About 25 years ago I became interested in geometric configurations of points and lines. It is known that (n_3) configurations exist if and only if $n \geq 3$. Branko Grünbaum[24] asked for which parameters n there exists an (n_4) configurations. He settled almost all the cases. Nowadays, only the case $n = 23$ remains open. Leah Berman, Gábor Gévay and I[25] asked the same question for any (n_k) configuration. For increasing k, the number of unsolved cases is finite but increasing. Almost every gap requires a new technique. The interplay between geometry and graph theory is fascinating.

– Ever since I learned that a protein chain may be designed in such a way that it is folded in the shape of a tetrahedron, I became interested in polyhedral self-assembly. The basic problem was originally modeled by Eulerian graphs. We turned it into a model of one-face embeddings of a graph on a closed surface[26]. There are many interesting questions in the extended case when the polyhedral edges need not be covered by exactly two segments of proteins, or if more than one chain is involved in the self-assembly process.

– A lattice is a well-known algebraic structure that admits a combinatorial description via the Hasse diagram of the corresponding poset. Roughly speaking, noncommutative lattices are similar algebraic structures where the

[21] P. Flajolet and R. Sedgewick, *Analytic Combinatorics*, Cambridge University Press, 2009

[22] A. Björner, M. L. Vergnas, B. Sturmfels, N. White, and G. M. Ziegler, *Oriented matroids*, Cambridge University Press, 1999.

[23] See `https://barabasi.com/f/622.pdf`.

[24] B. Grünbaum, *Configurations of Points and Lines*, (Graduate Studies in Mathematics), New ed. Edition, American Mathematical Society, 2009.

[25] L. W. Berman, G. Gévay, and T. Pisanski, *Connected geometric (n_k) configurations exist for almost all n*, The Art of Discrete and Applied Mathematics 4 (2021), #P3.14.

[26] G. Fijavž, T. Pisanski, and J. Rus, *Strong traces model of self-assembly polypeptide structures*, MATCH Commun. Math. Comput. Chem. 71:1 (2014), 199–212.

[27] J. E. Leech, *Noncommutative lattices. Skew lattices, skew Boolean algebras and beyond*, Famnit Lectures/Famnitova Predavanja 4. Koper: University of Primorska Press, open access, 2020.

commutativity axioms are dropped and the absorption laws modified. Quasi-lattices[27] form one of the most interesting varieties of noncommutative lattices. I am looking for a faithful combinatorial description of quasi-lattices.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

Pisanski: I started my work in the triangle with vertices: Graphs, Groups, and Surfaces, as nicely depicted in the book by Art White[28] by the same title. This triangle roughly covers the areas of topological and algebraic graph theory. Later I became interested in configurations and abstract polytopes. Both of these subjects belong to the area of incidence geometries. Envision a tetrahedron with four vertices: Graphs, Groups, Surfaces, and Geometries. In my chapter *Bridges between Geometry and Graph Theory* with Milan Randić[29] and later in my book with Brigitte Servatius *Configurations from a graphical viewpoint*[30] I was trying to explore the edge between Geometry and Graphs. Various symmetric classes of polyhedra or polycyclic configurations reside on the edge between Geometry and Groups. In the other direction, groups with their subgroups give rise to coset geometries with underlying incidence geometries. Polyhedral realizations of maps can be viewed on the edge between Geometry and Surfaces, etc. What I would like to see in the future is this tetrahedron filled with contents and perhaps consider higher-dimensional simplices by adding some more vertices, such as Linear Algebra (spectral graph theory), etc.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Pisanski: This is again a philosophical question. Research in mathematics is dynamic. What seems to be important this decade may become obsolete next decade. In my view, solving difficult open problems is more important than anything else. However, there is no objective criterion that would enable us to rank the importance of topics. Moreover, there is definite prejudice against discrete mathematics and combinatorics in particular. I am sure that combinatorics would be better off if discrete mathematics were considered as a separate field. Even nowadays, in many circles outside discrete mathematics, combinatorics and graph theory, in particular, are not considered mainstream. This hurts the field when competing for grant money.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Pisanski: Well, I would say all good mathematics is applied. Here, the meaning of the word *applied* is quite general: applications may be close or far away. They may be within mathematics or outside mathematics. The more applications a theorem has, the more important it is. On the other hand, intellectual activities outside mathematics may be a wonderful source of questions, problems, conjectures, and theories. About half of my work is on various applications, mostly in discrete mathematical chemistry. My most cited papers are published in journals like Journal of Mathematical Chemistry, Chemical Physics Letters, Periodicum Biologorum, Nature Biotechnology, Croatica Chemica Acta, or Journal of Chemical Society - Faraday Transactions, with co-authors coming from chemistry, physics, and synthetic biology. However, not all applied mathematics is good. Some of it may be really bad. Sometimes trivial mathematical results with no real applications are published in dubious journals without proper mathematical refereeing, under the umbrella of applied mathematics.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Pisanski: Hard work. Broad and deep education, an advisor with "good chemistry". Keep asking questions, learn to explain your work to others. Attack a difficult problem. If you cannot solve it completely, solve a special case. Learn to work alone and also as a member of a team. Avoid traps in predatory journals and conferences. A single publication in a jour-

[28] A. White, *Graphs, Groups, and Surfaces*, North-Holland Publishing Company, 1973.

[29] T. Pisanski and M. Randić, *Bridges between geometry and graph theory*, Geometry at work 53 (Mathematical Association of America Note), 2000.

[30] T. Pisanski and B.Servatius, *Configurations from a graphical viewpoint*, Birkhäuser Basel, 2013.

nal with dubious reputation may have a hugely negative impact on your career.

Mansour: Would you tell us about your interests besides mathematics?

Pisanski: Unfortunately, I am not very persistent and my interests change constantly. As a child, I enjoyed playing with model trains, reading books, and collecting post stamps. All these things are getting obsolete and hard to pass to the next generations. The thrill in collecting stamps was getting hold of letter envelopes and properly detaching stamps on them. Nowadays, no child is interested in collecting stamps, since stamps are no more around us. Same with train models. When I was young, traveling in Europe or Slovenia was done by train. Nowadays, it is done by car, bus, or airplane, and trains no longer carry their sentimental appeal. Nowadays, the generation of my grandchildren do not read anymore, they tweet instead.

Most of my interests are in one way or another related to mathematics.

I am interested in the history of mathematics, mostly related to Slovenia. I am interested in mathematics and art. For instance, I designed a model of Tucker's group of genus two[31] that was later realized as a sculpture by DeWitt Godfrey and Duane Martinez. A detail can be seen on the cover of the journal *Ars Mathematica Contemporanea*. I am interested in the mathematical and computational aspects of genealogy and heraldry. I am also fond of calligraphy. I collected a decent library on these subjects.

I always wanted to improve the position of discrete mathematics in contemporary society. That is why I was active in several societies. For many years, I was the Slovenian delegate at the International Mathematical Union and the European Mathematical Society (EMS), I am still active in the EMS.

I also helped found the Slovenian Discrete and Applied Mathematics Society, the journal Presek, covering mathematics, physics, and astronomy intended for pre-college students and teachers. I am a founding co-editor of two mathematical research journals: Ars Mathematica Contemporanea and The Art of Discrete and Applied Mathematics. For four terms I was on a panel for the European Research Council Advanced Grants. I was also the President of the Organizing Committee of the 8th European Congress of Mathematics that took part in Portorož, Slovenia. With such activities that present mathematics in Slovenia to the international mathematical community, we try to diminish the negative effects of wild globalization and replace the increasing brain drain by balanced international mobility.

Mansour: In one of your early works[32], in the 1980s, you calculated the genus of the cartesian product of any pair of connected, bipartite, d-valent graphs using a method later called *the White–Pisanski method*. Would you tell us about the main ideas behind this method?

Pisanski: I think the term the White-Pisanski method comes from the book *Topological Graph Theory*. Here is some background. In 1985, Tory Parsons, Wilfried Imrich and I organized a summer school on Algebraic and Topological Graph Theory[33] in the Interuniversity Center in Dubrovnik. Both Tom Tucker and Art White were speakers there. Tom brought with him a manuscript for the book he was writing with Jonathan Gross. Later he sent me the updates that I used for my graduate course in Ljubljana. The basic idea is charming and has been used already by Ringel (1955) and independently by Beineke and Harary (1965) to determine the genus of the hypercube graph Q_d, by constructing an appropriate quadrilateral embedding. The crucial part of the construction is the inductive step where one views $Q_d = Q_{d-1}\Box K_2$ as two disjoint copies of Q_{d-1}, joined by a perfect matching. One may visualize this construction performed on the quadrilaterally embedded copies of Q_{d-1}, equal but with opposite orientation. By removing a facial two-factor in each copy and replacing it with a collection of prismatic tubes, carrying as sides the perfect matching the step is completed. Art White used a generalization of this approach to compute the genus of the cartesian product of even cycles, thereby establishing one of the main tools for investigating the genus of

[31] T. W. Tucker, *There is one group of genus two*, J. Combin. Theory Ser. B 36:3 (1984), 269–275.
[32] T. Pisanski, *Genus of Cartesian products of regular bipartite graphs*, J. Graph Theory 4:1, (1980) 31–42.
[33] `https://www.fmf.uni-lj.si/~mohar/Bled/Dubrovnik85/Dubrovnik85.html`.
[34] A. T. White, *On the genus of a group*, Trans. Amer. Math. Soc. 173 (1972), 203–214.

various families of groups[34]. My main contribution was really minor. Namely, both K_2 and even cycles are bipartite, regular graphs. I noticed that the method devised by Ringel[35] and expanded by White[36] and his students and coauthors worked well because the edge of K_2 can be colored with one color and the edges of an even cycle can be colored with two colors. I extended their method to cartesian products of regular bipartite graphs of valence higher than 2. I managed to adopt this method to nonorientable surfaces[37]. It has then been exploited in a series of papers for finding optimal or near-optimal embeddings of various families of graphs with applications to groups[38,39].

Mansour: You are one of the founding members of *the International Academy of Mathematical Chemistry* (IAMC). What are the main objectives of the Academy? What are its highest achievements so far?

Pisanski: The IAMC was founded in Dubrovnik, Croatia, as a showcase of the Zagreb school of Mathematical Chemistry that spread throughout the world, in particular, to other parts of former Yugoslavia, such as Serbia and Slovenia. The main purpose of the Academy is to promote and develop the field of discrete mathematical chemistry in which chemical graph theory plays a prominent role. Originally, this field was regarded with considerable skepticism in certain circles of mathematics and chemistry. Several members of the IAMC have made a significant contribution to science. The first president, Alexandru Balaban with the discovery of his 10-cage is a well-known pioneer of graph theory. One of the founding members, recently deceased Nenad Trinajstić, wrote a fundamental monograph Chemical Graph Theory, that chartered this subfield of mathematical chemistry already in 1983. The initiator of the Academy was Milan Randič, who is a person with numerous excellent ideas. For instance, in 1993, he and Douglas Klein[40], one of the four current Vice-Presidents of the IAMC, wrote a paper that introduced the concept of resistance distance in graphs. The paper received wide attention both in mathematics (over 200 citations in MathSciNet) and outside (over 900 Web of Science citations). The editor of *Journal of Mathematical Chemistry* is Paul Mezey and the editor of *MATCH Communications in Mathematical and in Computer Chemistry* is Ivan Gutman. They are both members of the academy, as are many members of the corresponding editorial boards. The discovery of buckminsterfullerene resulted in our member, the late Harry Kroto[41], being awarded the Nobel Prize, spurring research of fullerenes and establishing once and for all the importance of graphs both in mathematics and theoretical chemistry. An outstanding expert on the theory of fullerenes, Patrick Fowler[42], FRS, is also an active member of the IAMC. Another important achievement, mostly due to the notable mathematical chemist Ivan Gutman, is the introduction of the code 05C92 - Chemical Graph Theory in the 2020 MSC.

Since the generation of founding members is diminishing and is less active, the Academy has to solve the problem of smooth generation change. Already in 2012, the untimely passing of the founding Secretary, Ante Graovac who was also the main organizer of the annual MATH/CHEM/COMP meetings in Dubrovnik presented a severe blow to the Academy. Unfortunately, the Covid crisis almost blocked its functioning in the past couple of years. In order to improve the situation, we are establishing a new headquarters of IAMC at the University of Primorska and are about to launch a new journal with the title *Discrete Mathematical Chemistry* and with Patrick Fowler, Klavdija Kutnar and myself as Editors-In-Chief. The journal will be closely associated with the Academy.

Mansour: You were among the prize winners in the IMO in 1967. What was the role of this achievement in your research career? Do you think that mathematics competitions in-

[35] G. Ringel, *Über drei kombinatorische Probleme am n-dimensionalen Würfel und Würfelgitter*, (German) Abh. Math. Sem. Univ. Hamburg 20 (1955), 10–19.

[36] M. Jungerman and A. T. White, *On the genus of finite abelian groups*, Eur. J. Comb. 1 (1980), 243–251.

[37] T. Pisanski, *Nonorientable genus of Cartesian products of regular graphs*, J. Graph Theory 6:4 (1982), 391–402.

[38] T. Pisanski and A. T. White, *Nonorientable embeddings of groups*, European J. Combin. 9:5 (1988), 445–461.

[39] T. Pisanski and T. W. Tucker, *The genus of low rank Hamiltonian groups*, Discrete Math. 78:1-2 (1989), 157–167.

[40] D. Klein and M. Randič, *Resistance distance*, J. Math. Chem. 12 (1993), 81–95.

[41] `https://www.nobelprize.org/prizes/chemistry/1996/press-release/`.

[42] P. W. Fowler and D. E. Manolopoulos, *An atlas of fullerenes*, Dover books on Chemistry, 2007.

spire young students for a successful research career?

Pisanski: Mathematical competitions represent a turning point in my approach towards mathematics. In 1964 an unexpected success in winning the first prize in the Slovenian math competition - the only one with a perfect score - triggered a positive feedback loop in my mind. I enjoyed winning by solving difficult problems, I enjoyed learning new material on my own. I liked challenges. I also learned from others. My motivation for learning was to acquire new methods to solve advanced problems that I could not solve before. In particular, I enjoyed training for an IMO team selection. We were stationed in a sports center in the outskirts of Belgrade, together with best athletes of Yugoslavia. Lectures on various topics, such as number theory, geometry, combinatorics, etc., were given by university professors and were followed by problems, some from previous IMOs. We learned some topics very well, e.g. mathematical induction and the pigeonhole principle.

Mansour: You are a member of the Ethics Committee of the EMS since 2010. What are the main ethical issues today within the mathematical community?

Pisanski: I was a founding member of the Ethics Committee in 2010. In 2012 we produced a document *the EMS Code of Practice* which addresses several ethical issues concerning publishing research in mathematics relating to authors, referees, publishers, and editors. The Committee also considered cases concerning plagiarism and other types of academic dishonesty and misconduct. Unfortunately, there are several forms of unethical behavior that are impossible to address. For instance, blacklisting predatory journals and publishers that would warn young researchers from falling into the trap practically disappeared from the internet due to the risks of expensive lawsuits. There is a respected journal that was run by a math department of a respected university. The editors would assign newly arrived manuscripts to graduate students, who would present them at a seminar. Anyone present at the seminar could take an unfair advantage, in particular, if the paper was later rejected. The rise of Open Access journals in connection with Article Processing Charge publishing model and Impact Factor manipulation represents a high risk of unethical practices. Money becomes a handy means for publishing anything in a high-impact journal. Elementary statistics show a dangerous trend. For instance, Journal of the European Mathematical Society used to be ranked among the top 10 journals on the WoS. This year it dropped below 20. This means that this year several journals learned the game of impact factor manipulation.

Statistically, one can easily show the presence of several aspects of latent discrimination from say, gender, geographic area, the research area. For instance, the percentage of discrete mathematics articles published in leading, general-purpose mathematical journals is much lower than the percentage in all publications.

Infiltration of non-mathematics in mathematics represents a serious problem and has negative impact on financing research in genuine mathematics. The problem is pertinent, since some fields, such as physics, chemistry, and biology are used to impact factors ten or twenty times higher than mathematics. If a non-mathematician competes with a mathematician for the same money and citations are considered as a ranking criterion, it may happen that a mediocre non-mathematician can seemingly outperform the best mathematicians. It is unacceptable that non-mathematicians determine what is mathematics. Some authors and journals that are non-essential for mathematics, according to MathSciNet or ZbMath, are ranked highly in WoS and Scopus.

Mansour: You have supervised around fifteen students in their Ph.D. thesis. What do you think about the importance of working with Ph.D. students and passing knowledge to them? Do you follow your students after they complete their thesis?

Pisanski: I consider supervising students and postdoctoral a great privilege of my career. According to the Math Genealogy Project, I have 16 Ph.D. students and 86 descendants. I am proud that several of my descendants are more successful than me. In 1981 I was the 23rd mathematician awarded a Ph.D. from a Slovenian University, the first one in discrete mathematics. Nowadays, almost 40%

of research mathematicians in Slovenia work in discrete math. I also encouraged our best students to complete their studies or post-docs training abroad in countries such as USA, Canada, United Kingdom, Austria, Australia or New Zealand. I enjoyed teaching undergraduate and graduate courses at various universities at home, in Croatia, Italy, Austria, USA, Canada, and New Zealand. I also participated as a lecturer in various summer schools and served as external examiner in several other countries.

Mansour: You have some papers on *the Wiener index of a graph*. What is the primary motivation for studying such parameters in graphs?

Pisanski: The original motivation comes from mathematical chemistry where certain chemical properties of a class of molecules are well correlated with a selected graph invariant, such as the Wiener index of the corresponding molecular graph. Graph invariants alias topological indices are special molecular descriptors that play a crucial role in modern chemistry and biochemistry. My first work on this topic was a linear time algorithm for computing the Wiener index of a tree that I developed together with my former student Bojan Mohar[43] in 1988. Browsing through the Web of Science, one can find over 2500 papers related to the Wiener index in journals from a variety of scientific disciplines, mostly covering chemistry, but also about 350 covering pure mathematics, I was pleasantly surprised by the revival of interest in our algorithm that found applications in large network analysis. I learned that the normalized Wiener index serves as a measure of structural virality[44] of online[45] diffusion and can be applied to networks with billions of nodes.

Mansour: One of your research interests is *Computational heraldry and genealogy*. Would you tell us about this field and your related works?

Pisanski: These topics are part of my general interests. When I began searching for my ancestors, the use of a computer became indispensable. Obviously, genealogy, graph theory, and network theory have many things in common. Computational aspects of genealogy are mostly due to the fact that storing and efficiently manipulating large sets of data comes naturally. There are also questions about the visualization of genealogical data. I wrote several computer programs. I even set up a web page for Slovenian genealogy through which people would communicate and through which I discovered and established contacts with some relatives that live in the USA.

Heraldry is all computational. Blason represents the oldest programming language for describing specialized graphics. It is much older than, say, postscript. It is formal and has several nice features: subprograms, recursion, default values, etc. Once I used it in an elective course, where my students had to implement an interpreter that would read blason and output a drawing of the corresponding coat of arms.

At some point, I included computational genealogy and heraldry as part of a project proposal that was unfortunately not funded. I am not sure if I will ever have time to turn my interests into research.

Mansour: You also have some interests in the history of mathematical sciences. Which period of math history are you interested in more? Besides research papers, reviews and surveys, ECA also publishes historical articles and biographies of mathematicians. How do you find this unusual mixture of research and history of mathematics?

Pisanski: I think you are doing the right thing. If we, mathematicians, do not care about our own history, who else will? In particular, this is true for discrete mathematics and for small countries. I became fascinated by Slovenian mathematician Jurij Vega (1754–1802). In 2002 I was coordinating the events commemorating the 200th anniversary of Vega's death, and two years later much more prominent festivities on the 250th anniversary of his birth that included a concert, a speech given by the President of Slovenia, a gun salute, an international symposium and a bilingual book on recent discoveries about

[43] B. Mohar and T. Pisanski, *How to compute the Wiener index of a graph*, J. Math. Chem. 2:3 (1988), 267–277.

[44] E. Lucherini, M. Sun, A. Winecoff, and A. Narayanan, *A Simulation Tool to Study the Societal Impact of Recommender Systems*, arXiv:2107.08959, 2021.

[45] S. Goel, A. Anderson, J. Hofman, et al., Management Science 62:1 (2016), 180–196.

[46] `https://www.dlib.si/details/URN:NBN:SI:SPR-U1OF1I43?&language=eng`.

Vega's life, entitled *Baron Jurij Vega and his times*[46]. Approximately at that time, Marko Petkovšek and I found out that the Lah numbers are named after a Slovenian mathematician Ivo Lah (1896–1979). I wrote Lah's biography[47] for MacTutor, history of mathematics.

Mansour: One of your great works at the intersection of convex/discrete geometry and combinatorial structures, co-authored with Marston Conder and Isabel Hubard, is *Constructions for Chiral Polytopes*[48]. Would you elaborate on it by emphasizing the combinatorial side of it?

Pisanski: My role in this paper was more catalytic. Namely, a decade earlier, in 1996, I first met Asia Ivić Weiss, the last student of Coxeter and advisor of Isabel Hubard. From her invited talk in Zagreb and discussions that followed, I learned that what geometers called an abstract polytope of rank 3 is almost the same thing as what we in topological graph theory called a 2-cell embedding of a graph in a closed surface or a map on a surface. Moreover, regular and chiral polytopes are almost the same thing as generalizations of regular and chiral maps. My goal was to bring these communities who spoke different languages together and I consider our paper as one of the key steps in achieving this goal. The main result of the paper is the first computer-assisted construction of a finite rank 5 chiral abstract polytope. It represents a superb synergy of the geometric expertise of Isabel Hubard and the low index subgroup approach mastered by Marston Conder. However, each abstract polytope is a ranked poset that can be represented as what we called a *link figure* of its Hasse diagram. Only when we turn the focus from symmetry to combinatorics, some interconnected layer graphs appear that make the structure, first produced by the computer, understandable to humans, without any need for further computations. My modest contribution was mostly in these, combinatorial aspects, understandable to humans.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Pisanski: Although I am not an expert, I find enumeration an important aspect of combinatorics. I dedicated some papers to enumeration. Even if the emphasis is elsewhere, you cannot go wrong by computing the initial terms of the series that enumerates a newly established structure. If the results indicate that the series is already covered by OEIS[49], the problem of finding a bijective proof emerges. However, if the series is new, one is faced with a natural and sometimes challenging problem of finding a closed-form solution for it.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Pisanski: Ever since I was preparing for the mathematical olympiad, I acquired some generic techniques for approaching a problem: analogy, generalization, simplification, and thinking out of the box. When I first considered graph bundles, I used analogy. A lot of theory of fibre bundles from algebraic topology carries over to discrete settings. On the other hand, graph bundles generalize covering graphs on the one side and cartesian products on the other. It is therefore natural to check which properties of the former are valid for the latter. Thinking out of the box comes easier if co-authors have different expertise. In a 2017 paper that I wrote with Simona Bonvicini[50] we successfully combined our complementary knowledge to characterize hamiltonian I-graphs. We used two ideas. First, we associated a certain quartic graph with a cubic graph with a 1-factor. Suddenly, we realized that in the case of I-graphs the associated quartic graph can be expressed as a graph bundle. This was certainly a "eureka moment" for that paper. The rest of the proof was just a careful albeit nontrivial execution of the program.

Throughout my career, I have met many smart individuals outside discrete mathematics with whom I have had the privilege to co-

[47] `https://mathshistory.st-andrews.ac.uk/Biographies/Lah/`.

[48] M. Conder, I. Hubard, and T. Pisanski, *Constructions for chiral polytopes*, J. London Math. Soc. 77(1) (2008), 115–129.

[49] `https://oeis.org`.

[50] S. Bonvicini and T. Pisanski, *A novel characterization of cubic Hamiltonian graphs via the associated quartic graphs*, arXiv:1508.01865.

operate, ranging from different fields of mathematics, such as operator theory, linear algebra, group theory, universal algebra, and geometry, as well as outside mathematics, for instance, computer science, physics, chemistry, synthetic biology, architecture, linguistics, library science, history, engineering, and art. Such a collaboration usually enhanced my thinking out of the box.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Pisanski: A long time ago I started with computations of genus of cartesian products of graphs. This approach was then expanded to finding efficient embeddings of Cayley graphs for various groups such as abelian- or hamiltonian groups. The presence of factors that are cycles of length 3 makes the problem extremely difficult and there is no hope of finding a complete solution, even for elementary abelian groups. Nevertheless, whenever I return to the problem, usually with my colleagues, we find new perspectives and fresh avenues of research to explore.

Mansour: My last question is philosophical: have you figured out why we are here?

Pisanski: Thank you, this is a very important and interesting question! I first came across this question in puberty, in high school. I was encouraged by the rational answer *Cogito ergo sum*, provided by René Descartes to a simpler question: *Are we here*? That led me to study the works of old philosophers. However, eventually, it became clear to me that there is no universally convincing answer insight, an answer that would not depend on faith or at least on the ideology. Quite honestly, I do not know why we are here. Even if I knew, I would not be sure how to argue my case rationally.

Mansour: Professor Tomaž Pisanski, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.