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HOLONOMIC SEQUENCES AND MODULAR FORMS: ALGORITHMIC SNAPSHOTS

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The study of holonomic functions (resp. sequences) satisfying linear differential (resp. difference) equations with polynomial coefficients has roots tracing back to the time of Gauss (at least). Tools to assist this study, including methods from computer algebra, have become fundamental for the modern theory of enumerative combinatorics; see, e.g., the work of Stanley and Zeilberger.

Also tracing back to the time of Gauss (at least) are highly non-holonomic objects: modular functions and modular forms with q-series representations arising, e.g., as generating functions of partitions of various kinds. Using computer algebra, the talk present some snapshots to illustrate how these two different worlds can be connected. Applications concern partition congruences, Fricke–Klein relations, irrationality proofs a la Beukers, or approximations to pi studied by Ramanujan and the Borweins. As a major ingredient to a "first guess, then prove" strategy, a new algorithm for proving differential equations for modular forms is used.

The results presented arose in joint work with Silviu Radu (RISC).