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BRESSOUD'S CONJECTURE ON THE ROGERS-RAMANUJAN IDENTITIES

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The Rogers–Ramanujan identities are a pair of infinite series-infinite product identities that were first proved by Rogers in 1894 and rediscovered by Ramanujan a few years later. Over the past decades, these identities and identities of similar types have found applications in many areas.

The most notable combinatorial generalization of the Rogers-Ramanujan identity is attributed to Gordon and is commonly referred to as the Rogers-Ramanujan-Gordon identity. In 1980, Bressoud put forward a conjecture for a more general partition identity that implies many classical results in the theory of partitions, such as Euler's partition theorem, the Rogers-Ramanujan-Gordon identities, and the Andrews-Göllnitz-Gordon identities and so on. Bressoud's conjecture depends on several parameters, and here we simply stated as $A_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)=B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ for $j=0$ or 1, where the function $A_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ counts the number of partitions with certain congruence conditions and the function $B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ counts the number of partitions with certain different conditions. Bressoud's conjecture was known in some special cases. The general case for $j=1$ was recently resolved by Kim.

In this talk, we present an answer to Bressoud's conjecture for the case $j=0$, resulting in a complete solution to the conjecture. It is somewhat unexpected that overpartitions play a crucial role in this regard. We first introduce a new partition function $\overline{B}_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ which could be viewed as an overpartition analog of the partition function $B_j(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ introduced by Bressoud. By constructing bijections, we show that there is a relationship between $\overline{B}_1(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and $B_0(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and a relationship between $\overline{B}_0(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$ and $B_1(\alpha_1,\ldots,\alpha_\lambda;\eta,k,r;n)$. Based on these two relations, we confirm Bressoud's conjecture for $j=0$. Besides, our approach leads to overpartition analogs of Bressoud's conjecture, which cover a number of overpartition analogs of classical theorems in the theory of partitions. The generating functions of overpartition analogs of Bressoud's conjecture are also obtained with the aid of Bailey pairs. The overpartition analogs explored in this study are not merely a matter of expansion or refinement, but rather are a crucial and fundamental component in our solution to Bressoud's conjecture regarding ordinary partitions.

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