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# THREE-EDGE-COLORING PROJECTIVE PLANAR CUBIC GRAPHS: A GENERALIZATION OF THE FOUR COLOR THEOREM

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We prove that every cyclically 4-edge-connected cubic graph that can be embedded in the projective plane, with the single exception of the Petersen graph, is 3-edge-colorable. In other words, the only (non-trivial) snark that can be embedded in the projective plane is the Petersen graph.

This implies that a 2-connected cubic (multi)graph that can be embedded in the projective plane is not 3-edge-colorable if and only if it can be obtained from the Petersen graph by replacing each vertex by a 2-edge-connected planar cubic (multi)graph. Here, a replacement of a vertex $v$ in a cubic graph $G$ is the operation that takes a 2-connected planar (cubic) multigraph $H$ containing some vertex $u$ of degree 3, unifying $G - v$ and $H - u$, and connecting the vertices in $N_G[v]$ in $G - v$ with the three neighbors of $u$ in $H - u$ with 3 edges. Any graph obtained in such a way is said to be *Petersen-like*.

This result is a nontrivial generalization of the Four Color Theorem, and its proof requires a combination of extensive computer verification and computer-free extension of existing proofs on colorability.