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SHARP LOWER BOUNDS FOR THE NUMBER OF MAXIMUM MATCHINGS IN BIPARTITE MULTIGRAPHS

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We study the minimum number of maximum matchings in a bipartite multigraph G with parts X and Y under various conditions, refining the well-known lower bound due to M. Hall. When $|X| = n$, every vertex in X has degree at least k, and every vertex in X has at least r distinct neighbors, the minimum is $r!(k-r+1)$ when $n \geq r$ and is $[r+n(k-r)](r-1)!/(r-n)!$ when $n < r$. When every vertex has at least two neighbors and $|Y| - |Y| = t \geq 0$, the minimum is $[(n-1)t+2+b](t+1)$, where $b = |E(G)| - 2(n+t)$. We have also determined the minimum number of maximum matchings in several other situations. We provide a variety of sharpness constructions.

These results are joint work with Alexandr V. Kostochka and Zimu Xiang.