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TREEWIDTH AND OUTERSTRING GRAPHS

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How is the treewidth of a graph related to its outerstring induced subgraphs? Trotignon conjectured that if a graph has large treewidth and is free of certain basic obstructions, then it must contain an outerstring induced subgraph of sufficiently large treewidth. We resolve this conjecture in the negative, providing a graph construction with arbitrarily large treewidth despite being free of basic obstructions and only containing outerstring induced subgraphs with treewidth bounded by an absolute constant. We show in particular that every induced subgraph of the constructed graph whose treewidth surpasses the aforementioned constant contains a sufficiently subdivided theta, which we prove is not an outerstring graph. Additionally, the construction we show can be modified to have arbitrarily large girth.

This is joint work with Maria Chudnovsky, Sepehr Hajebi, Sophie Spirkl, and Bartosz Walczak.