ICECA
International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 25-27, 2025

CRYSTAL SKELETONS: COMBINATORICS AND AXIOMS

SYLVIE CORTEEL

Department of Mathematics, University of California, Berkeley, CA, USA and CNRS, IMJ-PRG, Sorbonne Universite, France

Crystal skeletons were introduced by Maas-Gariépy in 2023 by contracting quasi-crystal components in a crystal graph. On the representation theoretic level, crystal skeletons model the expansion of Schur functions into Gessel's quasisymmetric functions. Motivated by questions of Schur positivity, we provide a combinatorial description of crystal skeletons, and prove many new properties, including a conjecture by Maas-Gariépy that crystal skeletons generalize dual equivalence graphs. We then present a new axiomatic approach to crystal skeletons. We give three versions of the axioms based on GL_n-branching, S_n-branching, and local axioms in analogy to the local Stembridge axioms for crystals based on novel commutation relations.

Joint work with Sarah Brauner, Zajj Daugherty, and Anne Schilling.