ICECA

International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 25-27, 2025

SHARPENINGS OF THE ERDŐS-KO-RADO THEOREM

GYULA O.H. KATONA

Rényi Institute, Hungarian Academy of Sciences, Budapest, Reáltanoda u. 13–15, 1053 Hungary

Let $[n] = \{1, 2, \ldots, n\}$ be our underlying set. $\binom{[n]}{k}$ will denote the family of all k-element subsets of $[n]$. A family $\mathcal{F} \subset \binom{[n]}{k}$ is called intersecting if any pair of its members have a non-empty intersection. The celebrated theorem of Erdős, Ko and Rado determines the maximum size of an intersecting family of k-element subsets: If $2k \leq n, \mathcal{F} \subset \binom{[n]}{k}$ is intersecting then $|\mathcal{F}| \leq \binom{n-1}{k-1}$.

Choose an integer $\ell \geq 2$ and take the following sum.

$$\sum_{1 \leq i < j \leq \ell} |F_i \cap F_j|.$$

If $\mathcal{F}$ is intersecting then every term here is at least 1, therefore the total sum is at least $\binom{\ell}{2}$. Does this weaker condition $\binom{\ell}{2} \leq \sum_{1 \leq i < j \leq \ell} |F_i \cap F_j|$ imply the upper bound of EKR? Much more is true for large n. Namely $\binom{\ell}{2}$ can be replaced by $\binom{\ell-1}{2} + 1$. However $\binom{\ell-1}{2}$ is not good enough, as a construction shows containing more than $\binom{n-1}{k-1}$ sets. A conjecture is posed that this construction is the best possible. A suprising connection to the Erdős Matching Conjecture is shown.

Joint work with Kartal Nagy.