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SPANNING TREES AND EULERIAN ORIENTATIONS

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An Eulerian orientation of an undirected graph is an orientation of the edges such that the in-degree and out-degree are equal at each vertex. Of course, this is is only possible if the degree of each vertex is even. In this talk we consider how many Eulerian orientations there are. This problem can also be defined for infinite lattices, which has importance in statistical physics, most famously in the study of water ice.

Our results include an estimate of arbitrary precision for graphs of sufficient expansion and degrees growing faster than $\log^8 n$. This employs a new tail bound on the cumulant expansion of a function of independent random variables. The estimate has a dependence on the inverse square root of the number of spanning trees, for which we do not have a heuristic explanation. Although this theorem is not proved for graphs of bounded degree, the same correlation with spanning trees still holds experimentally. This leads us to a new heuristic for sparse regular graphs that appears much more accurate than previous heuristics. For example, it is indistinguishable from experiment for real ice.

Joint work with Mikhail Isaev, Rui-Ray Zhang, and Iyer Tejas.