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## COMBINATORIAL APPROACH FOR LATTICE POINT COUNTING IN RATIONAL CONVEX POLYTOPES

GUOCE XIN

School of Mathematical Sciences, Capital Normal University, Beijing, China

The problem of counting lattice points in rational convex polytopes, which is equivalent to counting integer solutions to systems of linear Diophantine equations, is fundamental in both algebraic combinatorics and computational geometry, and constitutes the core subject of this report.

In computational geometry, Barvinok proposed a polynomial-time algorithm for counting lattice points in a rational polytope when the dimension is fixed in 1994. Ten years later, De Loera et al. implemented this algorithm by the C package LattE, which has now been developed as a “state of the art” algorithm in this area. However, lattice point counting in high dimensions still presents computational challenges.

In algebraic combinatorics, MacMahon’s partition analysis has become a common framework for handling linear Diophantine problems. The core problem is to compute the constant term of Elliott-rational functions, which includes lattice point counting and Ehrhart series computation as special cases.

This report will introduce our team’s work in this intersecting field: we introduce the geometric concept of “cones” into the algebraic combinatorics framework and systematically address the aforementioned core problem through the following three steps: i) decomposition into simplicial cones; ii) further decomposition into unimodular simplicial cones; iii) elimination of slack variables. We have made progress in each of these three steps. The new algorithm demonstrates clear advantages compared to the current mainstream tool LattE, and we have also discovered new classes of polytopes for which lattice point counting can be completed in polynomial time.