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Interview with Ken-ichi Kawarabayashi

Toufik Mansour

Ken-ichi Kawarabayashi completed his PhD at Keio University in 2001 under the supervision of K. Ota. During his career he was an Assistant Professor at Tohoku University (2003 - 2006) and an Associate Professor at the National Institute of Informatics (2006 - 2009) before being promoted to Professor. During his time there he held the position of Deputy Director General for two years. In October 2023 he also became a Professor at the University of Tokyo. His awards include the Takebe Prize (for an outstanding young Japanese mathematician) in 2001, the Kirkman Prize (from the Institute of Combinatorics and its Applications) in 2003, the IBM Japan Science prize in Computer Science in 2008, the Japanese Academy Medal in 2013, and the Fulkerson prize (jointly with Mikkel Thorup) in 2021. His areas of research include Discrete Mathematics and Theoretical Computer Science, Graph Theory, Graph Algorithms, Experimental and Scalable Algorithms, Graph Databases, Combinatorial Optimization, Machine Learning, and Quantum Computing. He has held positions on the editorial boards of the journals: Algorithmica, the Journal of Graph Algorithm and Applications, SIAM Journal of Discrete Mathematics, the Journal of Graph Theory, Discrete Mathematics and Theoretical Computer Science, Graphs and Combinatorics, and TheoretiCS.

Mansour: Professor Kawarabayashi, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Kawarabayashi: Many thanks for giving me this opportunity! I take "combinatorics" very broadly, including combinatorial optimization, combinatorial algorithms etc. To me, any "discrete" object is connected to combinatorics. In my case, anything to do with a "graph" is combinatorics. For example, many questions about a "matrix" can be translated into "graph" questions, and hence I consider these questions as a part of "combinatorics" as well.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Kawarabayashi: I always think that rapid progress of combinatorics influences a lot to the rest of mathematics. For example, Adam Marcus, Daniel Spielman, and Nikhil Srivastava[1] developed a combinatorial method which led to a solution of the Kadison–Singer problem, which was posed in 1959 and considered to be one of the most important conjectures in functional analysis. Another example is the Kannan–Lovász–Simonovits conjecture[2], which led to a lot of progress in high-dimensional geometry and probability.

Mansour: What have been some of the main goals of your research?

Kawarabayashi: There are two main goals: I would like to apply graph theory tools (including graph minor theory) to other areas

Toufik Mansour is a professor of mathematics at the University of Haifa, Israel. His email address is tmansour@univ.haifa.ac.il
[1] A. W. Marcus, D. A. Spielman, and N. Srivastava, *Interlacing families II: Mixed characteristic polynomials and the Kadison-Singer problem*, Ann. of Math. (2) 182:1 (2015), 327–350.
[2] Y. T. Lee and S. S. Vempala, *The Kannan-Lovász-Simonovits conjecture*, https://arxiv.org/pdf/1807.03465.pdf.

of mathematics and computer science. I am also interested in some graph theory questions which require computer search, including the Four Color Theorem and its generalizations.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Kawarabayashi: My father majored in design (not combinatorial designs, unfortunately). My mother plays violins. So my parents really wanted me to do some "creative" (artistic) job. When I was a kid, I did play violins (under my mother's influence), and I did learn paintings (under my father's influence), but my parents simply did not push me to pursue these, partly because I had no talent at all. Instead, "numbers" were more interesting for me.

Mansour: Were there specific problems that made you first interested in combinatorics?

Kawarabayashi: Definitely the Four Color Theorem[3]. I was really interested in understanding the proof of the Four Color Theorem when I was an undergraduate student. And yes, I am still hoping that we should get deeper understanding for the proof of the Four Color Theorem.

Mansour: What was the reason you chose Keio University for your Ph.D. and your advisor K. Ota?

Kawarabayashi: When I was a high school student, I was briefly interested in graph theory. At that time, Keio University was the only one that one could study graph theory in a mathematics department. So it was quite natural to me that I chose Keio University and Prof. K. Ota there.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Kawarabayashi: In most of the cases, my research is driven by problems or conjectures. I really want to contribute to some important problems/conjectures. But in some cases, my research is motivated by "methods and techniques" that I find very important. In this case, I would like to apply these methods and techniques to other areas/problems.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Kawarabayashi: Yes definitely. I start by conjecturing something, and try to show this. Indeed, if I cannot get any good intuition, most likely I cannot do anything. And my intuition is a rough path to lead to a complete proof.

Mansour: What three results do you consider the most influential in graph theory during the last thirty years?

Kawarabayashi: Graph minor theory by Robertson and Seymour[4]. This influences not only combinatorics but also logic and theoretical computer science (TCS). Almost all researchers in TCS know graph minor theory. This really deserves to be called a "Theory".

The strong perfect graph theorem[5]: Perhaps a solution of one of the most important questions in the last 20 years of graph theory.

Graph limit theory[6,7]: This bridges graph theory and statistics (and statistical physics). I feel that there are still a lot of things happening in the near future.

Mansour: What are the top three open questions in your list?

Kawarabayashi: My top would be a deeper understanding of the Four Color Theorem and its proof. In addition, I would like to see how metric embedding theory[8] that was developed in "pure" math helps to prove some big results in graph algorithms. Researchers like Assaf Naor[9] have tried to do this.

In addition, I would like to see other tools of combinatorics help to solve some big problems in some other areas of mathematics. I see that in theoretical computer science, but what else?

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

[3]F. Harary, *The four color conjecture. Graph theory*, Reading, MA: Addison-Wesley, 1994.

[4]N. Robertson and P. Seymour, *Graph minors. XIII. The disjoint paths problem*, J. Combin. Theory, Ser. B 63(1) (1995), 65–110.

[5]M. Chudnovsky, N. Robertson, P. Seymour, and R. Thomas, *The strong perfect graph theorem*, Ann. of Math. 164(1) (2006), 51–229.

[6]S. Chatterjee, *Basics of graph limit theory. In: Large deviations for random graphs*, Lecture Notes in Mathematics, volume 2197, Springer, Cham. 2017.

[7]L. Lovász and B. Szegedy, *Limits of dense graph sequences*, J. Combin. Theory, Ser. B 96:6 (2006), 933–957.

[8]I. Abraham, Y. Bartal, and O. Neiman, *Advances in metric embedding theory*, Adv. in Math. 228:6 (2011), 3026–3126.

[9]See, `https://web.math.princeton.edu/~naor/mat529.pdf`.

Kawarabayashi: As I said, I would like to have a deeper understanding of the Four Color Theorem. But at the same time, I would like to see how far we can push our understanding, using computer checks. So I would like to see a nice combination of mathematical arguments and computer checks.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Kawarabayashi: In a sense yes, but in a sense no. Sure, there is very exciting development in some areas, and one can call such an area "mainstream". But the hot area is not hot forever. So I still think that we should not care too much about "core" nor "mainstream" mathematics, but just focus on what we would work on, combinatorics.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Kawarabayashi: I was often asked "Are you a computer scientist or a mathematician?" My answer is always the same " I do not know, but I do not care." This probably applies to the distinction between pure and applied mathematics.

If you care about mathematical truth, and you do not care about applications of mathematics, you probably call every single application of mathematics as applied math. But in reality, practical things require an application of "applied math". So people from the practical side or outside mathematics cannot really see the difference between applied math and pure math. For them, it is just "mathematics".

Mansour: You also have experience in collaborating with industry. How has this interacted with your academic research? Would you say a few more words about any such cooperation?

Kawarabayashi: My experience in collaborating with industry is mainly in a theoretical aspect. Industries desperately require "correctness" and "guarantee" for their products, and only math can provide that.

This part does not require deep mathematics at all, but you do need to combine a lot of rather elementary mathematics. In addition, you really have to understand the model and parameters.

But once this is done, industries really appreciate your effort.

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Kawarabayashi: There are many pieces of advice I could offer, but let me focus on the "job". For many years, if you pursue a research career in mathematics, this means that you would like to get a job in academia. This is still the case, but it is changing. Some industries, like Microsoft, established some research labs for mathematics/computer science, and this seems very attractive. Indeed, they hire researchers who work on combinatorics/discrete math/graph theory.

This says that if you pursue you career in combinatorics/discrete math/graph theory, you have a lot of good opportunities for jobs (compared to "core" math, whatever it means).

Mansour: Would you tell us about your interests besides mathematics?

Kawarabayashi: I would really like to understand what/how my cats are doing. They know how to attract me and my family. They know how to get their favorite food from me. They hate to see me speaking online (they have no idea what is going on, because they do not see anything happening). Cats also teach me a lot. When you stick to one idea that does not work, you really have to throw away this idea and you have to start from the beginning. Cats teach me how to do that (how to forget about many things (mostly mess)).

Mansour: Japanese novelists are widely read globally. Haruki Murakami, Yasunari Kawabata, Yukio Mishima, Kenzaburō Ōe, Mieko Kawakami, and Kazuo Ishiguro write excellent novels that not only tell us about Japanese people and society but also touch on many essential aspects of human existence. Would you make our readers a list of Japanese novels?

Kawarabayashi: By the way, Kazuo Ishiguro is not Japanese (although he was born in Japan). Let me give a few more novelists. Both Yasushi Inoue and Ryotaro Shiba are widely known in Japan, but they mostly wrote about Japanese historical things.

Toyoko Yamazaki is also well known (I like a lot!). She wrote some stories based on actual events. For example, Shizumanu Taiyō (The Never-setting Sun in English) is based on the Japan Airlines Flight 123 accident (perhaps still the biggest airplane accident ever).

Mansour: You have delivered talks at numerous conferences, workshops, and seminars. How important do you find such activities?

Kawarabayashi: In most of talks I gave, I got several questions, and some of them were really unexpected. This is really important. Your view and others' are different. It is important to convey your view and your message to the audience. But at the same time, you should get more "diverse" feedback from the audience. Some people have totally different views from yours. This helps (and inspires) me a lot, because this is something I learn from the audience.

Mansour: *Neural networks* appear in some of your recent research articles. In the manuscript *What can neural networks reason about?*, coauthored with Xu, Li, Zhang, Du, and Jegelka[10], you say that *Neural networks have succeeded in many reasoning tasks.* Please tell us more about this? What should we understand from *reasoning*?

Kawarabayashi: In many applications of a Machine Learning, the function to be learned in neural networks may be viewed as having an underlying algorithmic structure, e.g., in relational reasoning about a set of objects, or forecasting physical simulations with Machine Learning. This paper has the following message: By studying which modeling strategies help learn algorithmic tasks for supporting optimization, we also, conversely, find how algorithmic concepts and knowledge can help design better Machine Learning models.

Mansour: Artificial Intelligence/Machine Learning (AI/ML) has been a very active research area in the last decade. Almost every month, we witness a new surprising advancement. Recently, machines can do many activities that we used to think only humans could do, such as writing novels, doing surgery, and composing music,... As far as I see, all these achievements mimic human activities. Do you think that machines will do something completely revolutionary or original in the future? I mean, creating something like mathematics, not like proving a theorem or writing a novel already done by humans?

Kawarabayashi: If you work on Machine Learning, you can immediately "learn" that all AI based methods depend on huge amount of data. The idea is that every single solution you need must be contained in this huge amount of data.

Sure, AI can now do writing novels, composing mucic, but when you see what AI creates, you would think that "I have seen this before" or something like this. This is simply because what AI creates is just a combination of known things. A mathematical proof could be just combination of known things, but the way to combine is not simple; in many cases, you have not seen such a combination before. In this case AI cannot do this.

In the future, AI can provide mathematical tools but using them to create something interesting is still mathematician's task.

Mansour: In one of your publications *One or Two Disjoint Circuits Cover Independent Edges Lovász–Woodall Conjecture*[11] you proved the *Lovász–Woodall Conjecture* namely that *if k is even or $G \setminus L$ is connected, then G has a circuit containing all the edges of L.* Would you explain your main approach used for the proof?

Kawarabayashi: This was done when I was an undergraduate student and I started writing it when I was a masters student. The approach is that by using previously known results, we can find a "path" P instead of circuit containing all the edges of L. Then I am trying to find a "circuit", but it was still hard. Thus I attempted to find either one or two circuits that contained all the edges of L, using the path P.

Mansour: A joint paper of you *From the plane to higher surfaces*[12], coauthored with Thomassen, has some interesting connections to the well-known 4-*color theorem.* Would you tell us more about this work?

Kawarabayashi: The Four Color Theorem

[10] K. Xu, J. Li, M. Zhang, S. S. Du, K. Kawarabayashi, and S. Jegelka, *What Can Neural Networks Reason About?*, `https://arxiv.org/pdf/1905.13211.pdf`.

[11] K. Kawarabayashi, *One or two disjoint circuits cover independent edges. Lovász-Woodall conjecture*, J. Combin. Theory, Ser. B 84:1 (2002), 1–44.

[12] K. Kawarabayashi and C. Thomassen, *From the plane to higher surfaces*, J. Combin. Theory, Ser. B 102:4 (2012), 852–868.

does not hold for other surfaces any more. Even worse, the five color theorem (every planar graph is 5-colorable) does not hold for other surface either. Having said these, the motivation of this paper is the following: For a graph G embedded in a surface (of Euler genus g) how "far" from the Four Color Theorem or the five color theorem? For the first case, we can only show that deleting $\sqrt{gn}$ vertices from G makes it 4-colorable. But for the second case, deleting only $1000g$ vertices from G makes it 5-colorable. We also provide this type of result for other problems as well.

Mansour: You have an interesting paper, *6-critical graphs on the Klein bottle*[13], co-authered with Král', Kynčl, and Lidický? Please tell us more about this work.

Kawarabayashi: A 6-critical graph G is not 5-colorable, but any proper subgraph of G is 5-colorable. From this notion, the Four Color Theorem says that there is no 5-critical planar graph. But this is not the case for the projective planar graphs, graphs embeddable in a torus or in a Kleinbottle. Therefore, researchers want to show that there are only finitely many 6-critical such graphs. This was done by Thomassen around 1990, but he did that for only projective planar graphs and for graphs embeddable in the torus. This paper does this for graphs embeddable in Kleinbottle. The proof also uses some computer checks.

Mansour: In a very recent paper *Rooted topological minors on four vertices*[14], co-authored with Hayashi, you characterized graphs G that contain no diamond on a prescribed set $\mathbf{Z}$ of four vertices, under some assumptions. Would you tell us about the motivation behind this work?

Kawarabayashi: This project was motivated by the famous Hajo's conjecture[15], which says that every graph without K_5-subdivisions is 4-colorable. This seems a far reaching generalization of the Four Color Theorem, and we wanted to obtain some partial results. This is what we did in this paper.

Mansour: Which ones do you consider the three most important applications of graph theory to other disciplines?

Kawarabayashi: Firstly, PageRank[16]. A random walk based method to compute the importance of each web page. This is perhaps one of the biggest achievements Google has ever made.

Graph Minor algorithm[4]: This has been used as a "blackbox" to obtain an efficient algorithm for many problems.

Szemeredi's regularity lemma[17]: This has been used for many places in the whole mathematics.

If I am allowed to add one more, Thomassen's proof[18] for the Jordan closed curve theorem, using Kuratowski's theorem. The Jordan closed curve theorem appears everywhere in mathematics.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Kawarabayashi: I have been working on some computational questions, both theoretically and practically. It turns out that enumerative techniques are very important in practice. More specifically, "pruning", which tries to avoid repetitions of computations is a key to obtain a scalable algorithm. More precisely, for most of the computationally hard problems (i.e., NP-complete problems), speeding up your algorithms by factor n, say, is not really possible. Instead, we are trying to get some 100 times speed up, by using enumerative techniques for pruning.

Mansour: How do we write Enumerative Combinatorics in Japanese characters? Does it have special meaning? Your name, Ken-ichi, means "wise one." Does your name affect your decisions?

Kawarabayashi: "数え上げ組合せ", where "数え上げ" is "enumeration" and "組合せ論" is combinatorics. This name is probably named after Stanley's famous book.

[13]K. Kawarabayashi, D. Král̆, J. Kynčl, and B. Lidický, *6-critical graphs on the Klein bottle*, SIAM J. Discrete Math. 23:1 (2008), 372–383.

[14]K. Hayashi and K. Kawarabayashi, *Rooted topological minors on four vertices*, J. Combin. Theory, Ser. B 158 (2023), 146–185.

[15]R. Diestel, *Graph theory*, Graduate rexts in mathematics, volume 173 (3rd ed.), Springer-Verlag, 2006, pp. 117–118.

[16]D. Sullivan, *What is google PageRank? A guide for searchers & webmasters*, Search Engine Land. Archived from the original on 2016, 2017.

[17]E. Szemerédi, *On sets of integers containing no k elements in arithmetic progression*, Acta Arith. 27 (1975), 199–245.

[18]C. Thomassen, *A link between the Jordan curve theorem and the Kuratowski planarity criterion*, Amer. Math. Monthly 97:3 (1990), 216–18.

My first name is "健一", which means that "healthy is first", not "賢一", which means "wise one". Sure, my parents are always right that my health (both physically and mentally) is the most important thing in my life.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Kawarabayashi: When I was young, I had worked on Hadwiger's conjecture[19]. Although I did not see myself solving this conjecture, I had been working on the next open case. This was still very hard, but Bjarne Toft told me about some related problem we could work on. It was quite interesting to me because the techniques I know can still work for this problem. However, there is one step I could not solve for a few weeks, and it turns out that a partial solution was done by Robertson, Seymour, and Thomas[20] on their proof of Hadwiger's conjecture for K_6-minor-free case. I just nailed their method, and eventually we got[21] the result "Any 7-chromatic graph has K_7 or $K_{4,4}$ as a minor".

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Kawarabayashi: I have been working partly with my postdocs and students to understand more about computer assisted proofs of the Four Color Theorem. We have been working on some generalizations of the Four Color Theorem for a few years. We have made pretty good progress but are not yet ready to publish.

Mansour: Japan is, in many aspects, a unique country in terms of its history, culture, cuisine, and tradition. People from other cultures often consider it an exotic destination. Does Japan also have a unique character for doing mathematics?

Kawarabayashi: I would say "no", but I might be biased. My way of doing math is perhaps different from "typical" researchers in Japanese math community. I am interested in graph theory/graph algorithms and their applications, so the area is very diverse (and it spans from math to computer science/physics). I have seen a lot of areas, and I really enjoy that. This kind of approach is not typical in Japan. The Math community in Japan tends to prefer some deep theorems/techniques and prefers to stay in one area.

Mansour: In a very recent short article[22] published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?" concluded that "Mathematical truths exist, and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

Kawarabayashi: This is totally my personal opinion. I think that if you say *invented*, this should mean that you show something a "deep" that affects others. So I would use *invented* for deep theory or for a solution to a difficult conjecture. If you say *discovered*, this should be somehow "surprising". If you find some surprising connections/bridges, I would call it *discovered*.

Mansour: Professor Kawarabayashi, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Kawarabayashi: Many thanks for this opportunity!

[19] H. Hadwiger, *Über eine klassifikation der streckenkomplexe*, Vierteljschr. Naturforsch. Ges. Zürich 88 (1943), 133–143.

[20] N. Robertson, P. Seymour, and R. Thomas, *Hadwiger's conjecture for K_6-free graphs*, Combinatorica 13(3) (1993), 279–361.

[21] K. Kawarabayashi and B. Toft, *Any 7-chromatic graph has K_7 or $K_{4,4}$ as a minor*, Combinatorica 25(3) (2005), 327–353.

[22] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.