ecajournal.haifa.ac.il
Enumerative Combinatorics and Applications
ECA **4:4** (2024) Interview #S3I7
https://doi.org/10.54550/ECA2024V4S4I7

Interview with Gyula O. H. Katona

Toufik Mansour

Gyula O. H. Katona completed his Ph.D. at Eötvös Loránd University, Hungary, in 1968 under the supervision of Alfréd Rényi. His research predominantly focuses on combinatorial set theory, and he has been twice awarded the Grünwald Prize, the Bolyai Society Prize for outstanding young mathematicians, in 1966 and 1968. Additionally, he received the Prize of the Hungarian Academy of Sciences in 1989. Currently, he holds the position of Research Professor Emeritus at the Alfréd Rényi Institute of Mathematics of the Hungarian Academy of Sciences. During his career, he has held visiting positions at the University of Göttingen (1974), the Colorado State University (1978-79), the Moscow Mathematical Institute of the Soviet Academy of Sciences (1979), the Ohio State University (1985), the University of California San Diego (1985-86), and the University of Illinois Urbana-Champaign (1993). He is a member of the European Academy of Sciences, an ordinary member of the Hungarian Academy of Sciences, and a Foreign member of the Bulgarian Academy of Sciences. Sciences. He has served as an editoral board member for the Journal of Statistical Planning and Inference, the European Journal of Combinatorics, Random Structures and Algorithms, and Studia Sci. Math. Hungar. (Editor-in-Chief, 1996-2006), and is presently on the editorial boards of Discrete Mathematics, Combinatorica, and Acta Mathematica Hungarica.

Mansour: Professor Katona, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Katona: I do not think I can give a precise definition. Although I prefer and use the term Combinatorics, the expression Discrete Mathematics describes the concept better. Those problems are considered when the elements of the studied structure are not continuous, but discrete. In other words, the number of elements is finite or at most countable infinite. Ok, the theory of Finite Groups is a counterexample.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Katona: For a long time Combinatorics was not considered a real science, only a collection of funny puzzles. A good example of this was when Erdős, Ko and Rado[1] discovered their now-celebrated theorem in 1938 they did not find it worth of publishing. Only 23 years later! Since the nature of combinatorics was quite different from the other areas of mathematics, there was no strong connection with the traditional mathematical disciplines. Therefore it was developing independently for a long time. The appearance of digital computers strengthened this progress. But it was quite natural that later these connections with the other areas slowly developed for the benefit of both combinatorics and the rest of mathematics. Unfortunately, my contribution is very mod-

Toufik Mansour is a professor of mathematics at the University of Haifa, Israel. His email address is tmansour@univ.haifa.ac.il
[1] P. Erdős, C. Ko, and R. Rado, *Intersection theorems for systems of finite sets*, Quart. J. Math., Second Series, 12 (1961), 313–320.

est because of my weak knowledge in other areas. But my papers on "continuous versions"[2] of extremal combinatorial problems were some small steps in this direction.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Katona: In the turn of the years 1944 and 1945 the Soviet Army was liberating Hungary from the German and Hungarian fascists. I was then less than 4 years old. During these operations, Soviet soldiers shot to death my parents on the street without any question or warning. A strange family took me home. Although I carried the bags of my parents with the documents, it was impossible to contact my relatives about the sad event. I stayed with the family for more than two months. The family had a daughter of age 7 and they engaged a private tutor for her. Once he asked an easy question concerning fractions. The girl did not know the answer, but I, who was playing around gave the correct solution. There were no comments, but I was very proud of myself: I knew something that a 3 years older girl did not. It is obvious that my father who was an engineer must have taught me something like this.

I arrived at my new home in March 1945. My mother's family, her parents, and her older sister, became my new family. My grandpa was very sick and died within a year. Therefore my grandmother and my aunt raised me up. Although my aunt did not finish the 5th grade, she was teaching me the love of numbers and logical thinking. I was a good pupil and I saved in my memory several cases when I surprised people with my solutions of "mathematical" problems.

However, the crucial point was when a friend of mine called my attention to the High School Mathematics and Physics Journal (founded in 1893). They had yearly point contests. I had to (try to) solve 8 difficult problems each month, much beyond the regular school problems. I enjoyed them very much. (To write down the solutions was less enjoyable.) At the end of the school year, they published my picture in spite of the fact that my score was not very good, but it was the best among the students going to "technical high schools". (Yes, I am a telecommunication technician, I had the license to repair radio and TV sets produced before 1960.) As a result of the practice, I finished at places 5 to 9 in the Hungarian National Olympiad in 1957, in my second year of high school. In the next year, I got a shared first prize with two others. It was great luck that Romania started the Series of the International Mathematical Olympiad in 1959. I was selected to be a member of the Hungarian team. Based on these results I decided to study mathematics instead of engineering. I entered the Eötvös University in Budapest.

Mansour: Were there specific problems that made you first interested in combinatorics?

Katona: In 1960 my classmate and friend, Domokos Szász (now a member of the Hungarian Academy of Sciences) and I solved a problem on tiling with dominoes, posed in the Matematikai Lapok (Mathematical Pages) and then started to generalize it. We arrived at the following conjecture: an $m \times n$ rectangle can be tiled with many copies of $1 \times k$ dominoes if and only if one of m and n is divisible by k. One morning still in the bed, half sleeping, a nice idea came to my mind: I found a proof using the roots of unity for a coloring. This was my first original mathematical idea. (We kept working on similar problems and published a paper only in 1971.[3])

Mansour: What was the reason you chose Eötvös Loránd University for your Ph.D. and your advisor Alfréd Rényi?

Katona: To give a true answer, first I have to describe the system of scientific qualifications in those days in Hungary. Adopting the Soviet system, the real first degree was the "candidate of sciences". It was a scientific degree of a higher level than the usual Ph.D. We were told that 6 good papers are needed. One could get a fellowship, but those people who had an academic job (or one in a research institute, like me) did not enter the system, just submitted their thesis when they thought it was sufficiently strong. Unlike in the Soviet Union where the universities did not have a Ph.D., in

[2] G. O. H. Katona, *Continuous versions of some extremal hypergraph problems. II*, Acta Math. Hung. 35 (1980), 67–77.
[3] G. O. H. Katona and D. O. H. Szász, *Matching problems*, J. Combin. Theory 10 (1971), 60–92.

Hungary the old Ph.D. was saved. However, if someone had the "candidate" degree, could automatically get the Ph.D. from his university. This situation under-valuated the Ph.D.'s issued by the universities. Very few people wanted to have the simple Ph.D. of a university.

My masters were Erdős, Rényi, and Turán. In 1968, I had a sufficient amount of results for a "candidate" thesis, but I still did not write a thesis, I did not feel an urge to do it fast. In the summer of 1968, Rényi obtained an invitation to spend the next spring semester in Chapel Hill, North Carolina. "But you can take a young man with you, as well." Rényi chose me. But the American university could pay me only if I had a Ph.D. The "candidate" degree was too slow, it was impossible to finish it within a couple of months. This is why I submitted a thesis for the university Ph.D. at Eötvös Loránd University. It contained only one paper, the solution of a problem of Rényi. I received the degree within 3 months.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Katona: Mostly the specific problem, but there are counter-examples. A couple of times a new model of a situation of real life.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Katona: Of course. And I am mostly right!

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Katona: I am reluctant to answer this question because it will unveil how narrow my knowledge is. On the other hand, if I name the *graph limits* of Lovász, you could object that this is a new theory, only related to combinatorics. But OK, let me say that the solution of the strong perfect graph conjecture (Chudnovsky, Robertson, Seymour, and Thomas[4]) and Keevash' proof of the existence of Steiner systems[5] are such influential results.

Mansour: What are the top three open questions in your list?

Katona: When I was young, I had two problems I wanted to solve. The first one was the Turán 4,3 problem (the maximum number of 3-element subsets of an n-element set if no 4 of them are within 4 elements). The other one was the generalization of the Erdős-Ko-Rado theorem for t-intersecting families. But I wanted to solve them later, when I have no administrative duty, no teaching, no students. And I was sure they would not be solved before that. In spite of this belief of mine the latter one was solved by Ahlswede and Khachatrian[6]. But the Turán problem will remain open for the years when I will be really old. A relatively new open problem of the Extremal Set Theory is the following one. A family of subsets of an n-element set contains no 4 members A, B, C, D such that A is a subset of $B \cap C$ while $B \cup C$ is a subset of D. It is easy to see that the two middle levels of the Boolean lattice satisfy the condition. One has to prove that this is asymptotically the best. This is called the *diamond problem.* The third problem is closer to design theory. Baranyai's theorem says that if $k|n$ then the family of all k-element subsets can be decomposed into subfamilies in such a way that each subfamily gives a partition of the underlying set. The open problem (Baranyai and Katona) would give a generalization for the cases when $k \nmid n$.[7]

Mansour: What kind of mathematics would you like to see in the next ten to twenty years as the continuation of your work?

Katona: The solutions to the problems given above, especially a nice general method to solve problems similar to the diamond problem.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Katona: Yes, there are, but I can tell you for sure only with a delay of 20 years.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the

[4]M. Chudnovsky, N. Robertson, P. Seymour, and R. Thomas, *The strong perfect graph theorem*, Ann. of Math. 164 (2006), 51–229.

[5]P. Keevash, *The existence of designs*, arXiv:1401.3665.

[6]R. Ahlswede and L. H. Khachatrian, *The complete intersection theorem for systems of finite sets*, European J. Combin. 18:2 (1997), 125–136.

[7]G. O. H. Katona, *Rényi and the combinatorial search problems*, Studia Scientiarum Hungar. 26 (1991), 363–378. (Page 370.)

relationship between so-called "pure" and "applied" mathematics?

Katona: I think there is only one mathematics. Some of its areas are close to a real-life model, some are farther. Moreover, it depends on your viewpoint: the most "applied" mathematics looks very theoretical in the eyes of an engineer. In my personal practice, the applied models raised beautiful theoretical questions. I would say that there are two sources of new mathematical problems. 1. The inner progress of the area. The problems solved trigger new problems: their combinations and generalizations. 2. Practical applications in other sciences or other branches of mathematics raise new problems.

Mansour: Can you share some memories from your participation in the 1959 IMO? What was it like back then compared to today's Mathematical Olympiads?

Katona: In those years (1959!) it was not common to travel to foreign countries at all. I personally did not know anybody who would have crossed the borders of Hungary. Only sportsmen and official delegations were travelling abroad. Therefore the mere fact that we went to Romania was a delightful experience. The mathematical contest was very much the same as now, but on a much smaller scale. only 7 countries were invited, the "European socialist countries", but one team consisted of 8 people (now 6). For some reason unknown to us, the Soviet Union was represented by 4 students, only. My results were pretty bad, yet I also contributed to the 2nd place of the Hungarian team. A big surprise was East Germany: the whole team got as many points as one Hungarian.

Mansour: As an advisor, you've influenced the careers of many students. What advice do you have for young mathematicians who are just starting their academic journeys?

Katona: I am sorry, but I do not have any general advice.

Mansour: Would you tell us about your interests besides mathematics?

Katona: Languages. I have passed official exams in English, Russian, and Polish languages. My German is full of mistakes but fluent. My Czech, Bulgarian, and Italian are childish. But I studied about 40 languages a little bit. Before going to a country I study its language at least 2 weeks prior to the trip. Of course, I forget it fast, but next time my knowledge comes back faster. When I am in the country, I can read Korean, Armenian, Hebrew, Arabic, and 6 Indian scripts. Now I am studying Chinese.

This hobby is still alive. But at younger ages, I had other ones, too.

Playing music. I played the violin. Professor Rényi and I played chamber music. He was the pianist. Later I played the recorder (Blockflöte) more by the influence of Baranyai who was a professional recorder player. It is much easier to pick up a recorder or a mouth organ and play than a violin which needs some preparation.

In my years of 40-44, I was a member of the amateur theater of the Polish Cultural Center (in Budapest). Our goal was to popularize the Polish literature. Once we even played in a real theater in Warsaw (Teatr Adekwatny).

Chess. When I was 13 or 14, I was a member of the team of Hungary of this age group. Some other members became great players, like István Csom, a FIDE grandmaster. On the one hand, I was not as good, on the other hand, I thought it was in conflict with mathematics, so I gave it up. But later I played in the teams of the University and the Institute.

Juggling. Influenced by Ron Graham and Peter Frankl I also juggled in a certain period. I was able to keep 5 balls throwing. (Caught, as well!)

Now, I can give one advice to young mathematicians: do not waste too much time with stupid hobbies.

Mansour: You are best known for your work in combinatorial set theory, and especially for the Kruskal–Katona theorem[8,9]. Would you please expand on this?

Katona: We have a family of m sets of size k. The *shadow* of the family consists of the sets which can be obtained by deleting one element from a member. That is, the shadow is a family of $k-1$-element subsets. Given k and m, what is the minimum size of the shadow? It seems to be obvious that the best is to choose the

[8] G. O. H. Katona, *A theorem of finite sets*, in P. Erdős and G. O. H. Katona (eds.), Theory of Graphs, Akadémiai Kiadó and Academic Press, 1968. Reprinted in Gessel & Rota (1987, pp. 381–401).

[9] J. B. Kruskal, *The number of simplices in a complex*, in R. E. Bellman (ed.), Mathematical Optimization Techniques, University of California Press, 1963.

family on the smallest possible underlying set. The Shadow Theorem (less modestly Kruskal-Katona) says that this is really the best. Nowadays there are much shorter proofs than the original ones, but there is no trivial one.

Mansour: You are also well known for your beautiful and elegant proof of the Erdős–Ko–Rado theorem in which you discovered a new method, now called Katona's cycle method[10]. Would you tell us more about this work?

Katona: The theorem of Erdős, Ko, and Rado claims that claims that if a family of k-element subsets of an n-element set (suppose $2k \leq n$) is intersecting (any two members have a non-empty intersection) then the number of sets is at most $\binom{n-1}{k-1}$ and this is equal to $\frac{k}{n}$ times the total number of k-element subsets. I got the idea that if this ratio holds for a substructure then an easy double counting will give the theorem for the whole structure. This substructure was the family of intervals (of length k) along a cyclic permutation. Let me call your attention to the similarity to the Baranyai-Katona[7] conjecture!

Mansour: *The Tuán number* is a recurring theme in many of your research articles. Could you explain the significance of this number in Extremal Combinatorics and the broader context of your work?

Katona: Turán's theorem[11] is the first theorem of Extremal Combinatorics. At least I start my courses with it. Then it is quite natural to ask the maximum number of edges under the condition of forbidding other graphs. At least according to my personal taste. Moreover, the Turán 4,3 problem is one of the oldest and most interesting open problems in Combinatorics.

Mansour: Besides combinatorics, your research interests include the theory of databases, search theory, and cryptology. What cross-disciplinary insights can be gained from these interactions?

Katona: Let me first tell you why I started to work in these "applied" areas. Professor Rényi was one of the (several independent) initiators of *Combinatorial Search*. Here an unknown element x is sought in a finite set by asking subsets if they contain x or not and x should be determined based on this information. Of course, only certain subsets can be used, and the mathematical problem is to minimize the number of questions. The model is really *statistical-information-theoretical*, but in the case when the next question cannot depend on the previous answers then this problem belongs to Extremal Set Theory. I was still an undergraduate student when I solved one of such problems of Rényi. This is why it became an important area for me. I felt it was my duty to continue the seminar of Rényi in the area, it is now more than 60 years old.

The reasons I started to work in the theory of databases were diplomatic-financial. The Hungarian Academy had another institute: Computer and Automation Institute. It had some groups of mathematicians working in applications for the industry. This Institute was much richer than ours. We started the cooperation to obtain moral support (real applications, not only useless theories!) and money for the Institute. We started a joint seminar on database theory. We found very nice and interesting extremal problems for matrices, very similar to problems of Extremal Set Theory. It also turned out that *secret sharing* (a chapter of cryptology) also needs the solution of extremal problems for matrices, similar to that of database theory.

The Ministry of Education offered large grants for innovative joint work with an industrial company. Until then the credit cards were identified only by their magnetic tape, but it was not safe, and could be easily copied. We developed a new method for the identification on the basis of a small picture of 3-dimensional nature, in cooperation with Hewlett-Packard Hungary. They have even built a working ATM based on the method. It was also patented. To prove the practical applicability we had to prove a nice combinatorial theorem.[12] Unfortunately, in the meantime the credit card companies started to add the chips and it turned out cheap and easy, our invention became meaningless. Yet, this work brought a

[10] G. O. H. Katona, *The cycle method and its limits*, Numbers, information and complexity (Bielefeld, 1998),129–141, Kluwer Acad. Publ, Boston, MA, 2000.

[11] P. Turán, *On an extremal problem in graph theory*, Matematikai és Fizikai Lapok (in Hungarian) 48 (1943), 436–452.

[12] L. Csirmaz and G. O. H. Katona, *Geometric codes* (in Hungarian), Alkal. Mat. Lapok 23 (2006) 349–361. see also: *Geometric cryptography*, in Proc. International Workshop on Coding and Cryptography, 2003.

large amount of money to the Institute. Also, this practical problem led us to solve an absolutely theoretical extremal problem.[13]

Mansour: Professor Katona: *When the lie depends on the target?*

Katona: An example of Combinatorial Search is the criminal investigation. There is a finite set of suspects, and one of them is the perpetrator x. An evidence determines a subset in which the x can be. Or e.g a witness is asked if x was taller than 180cm. The answer also gives a subset of the suspects. Of course, x should be found with the smallest number of such questions. Of course, there are many variants of this search model: there can be more elements to be found, their number is known or unknown, etc. One such variant is when the answers are not reliable. There is a strong branch of the theory of Combinatorial Search namely *search in the presence of a liar.* Here it is supposed that the number of erroneous (wrong) answers is at most e. Traditionally in these "liar" models all x's and question sets have the same role, their answers are wrong independently.

Suppose now that the witness is a relative of the perpetrator x. Then the answer is not reliable. Otherwise yes. This situation generated our new model for *search in the presence of a liar.*[14]

Mansour: In a recent paper *The domination number of the graph defined by two levels of the n-cube, II*[15], co-authored with József Balogh, William Linz, and Zsolt Tuza, you proved a conjecture on domination number $\gamma(G_k, \ell)$ for the case $\ell = 2$. What is a *domination number*? What about the results in cases $\ell > 2$? Would you describe this work a little bit more?

Katona: A *dominating set D* in a graph is a set of vertices such that every vertex is either in D or it has a neighbor in D. The *domination number* of the graph is the smallest size of the dominating sets. When Leila Badakhshian came to work with me from Iran and I saw that she had some results concerning the domination number, I tried to create a problem to work on which is related to her earlier work but partially belongs to the Extremal Set Theory. This was it: Suppose that $n > k > \ell$ and consider the graph whose vertex set consists of all k and ℓ-element subsets of an n-element set. Two vertices are adjacent if and only if one of them is a proper subset of the other one. Find the domination number of this graph. This innocent-looking problem turned out to be rather difficult. Although Zsolt Tuza joined the team, we were able to prove only estimates and formulated an asymptotic conjecture for the case of arbitrary k and $\ell = 2$.

Later Zsolt and I proved the conjecture, but it turned out that Balogh and Linz independently also proved it. We decided to write a joint paper. The case when ℓ is more than 2 seems to be as difficult as extremal problems for uniform hypergraphs, e.g. the Turán 4,3 problem.

Mansour: In your work, you have extensively used combinatorial reasoning to address important problems. How do enumerative techniques engage in your research?

Katona: I know that Enumerative Combinatorics is an important and interesting science. But somehow I did not meet it at my younger age. The enumerations used in Extremal Set Theory are rather easy. But there is a single paper where I used some heavy enumeration[16]. But I must acknowledge that the referee claimed that I should have used some known methods from probability theory instead of my tedious work. And it has one single citation.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Katona: Erdős called our attention to a problem asked in the famous Erdős-Ko-Rado paper: how many subsets can you choose from an n-element set if the intersection of any two sets has size at least t? It seemed obvious that the

[13] P. Frankl, M. Kato, G. O. H. Katona, and N. Tokushige, *Two-coloring with many monochromatic cliques in both colors*, J. Combin. Theory Ser B 103 (2013), 415–427.

[14] G. O. H. Katona and K. Tichler, *Search when the lie depends on the target*, in: Information Theory, Combinatorics, and Search Theory (In Memory of Rudolf Ahlswede) (H. Aydinian, F. Cicalese, Ch. Deppe, Eds.) LNCS 7777, Springer, 2013, 524–530.

[15] J. Balogh, G. O. H. Katona, W. Linz, and Z. Tuza, *The domination number of the graph defined by two levels of the n-cube, II*, European J. Combin. 91 (2021), 103201.

[16] G. O. H. Katona, *Testing functional connection between two random variables*, in: Prokhorov and Contemporary Probability Theory, (A. N. Shiryaev, S. R. S. Varadhan, E. L. Presman, eds) Springer, 2013, 335–348.

best construction (suppose that $n+t$ is even) is when all sets of size $\frac{n+t}{2}$ or more are taken. I was then 20, a second-year student, and spent all my free time through 3 months with trying to solve this problem. My starting idea was that it is sufficient to look at two levels, that is try to prove that the total number of sets (under the given conditions) of sizes i and $n+t-i-1$ ($i \leq \frac{n+t}{2}$) is the largest if all sets of size $n+t-i-1$ are chosen. If this is true for every i then we are done. Later it turned out that this was true, I was lucky!

In other words, if there are m sets of size i then they kill at least m sets of size $n+t-i-1$. Take an i-element member of the family. Its intersection with other members has at least t elements, therefore if $t-1$ elements are deleted from the i-element set, then the so the obtained $i-t+1$-element set still intersects the $n+t-i-1$-element members of the family, hence their complements cannot be equal to these members of size $n+t-i-1$. To summarize, if we take all $i-t+1$-element subsets of the i-element sets in the family, their complements are killed. It means that we only have to prove that the *deep shadow* of the family of all i-element sets is large enough. Here, of course, the condition that they are t-intersecting must be exploited.

Here I arrived at a new problem: there is a t-intersecting family of a-element sets, then the ratio of the size of b-shadow ($1 \leq b \leq t$) (the family of all $a-b$-element sets obtained by deleting b elements from a member) and the size of the original family of a-sets is the smallest for the following construction: all a-element subsets of a $2a-t$-element set. It was not difficult to prove this using the technics of the Erdős-Ko-Rado paper and some manipulations with the binomial coefficients. And the result was sufficiently strong for the desired purpose. I was lucky again.[17] This is how I met the shadows first.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Katona: No, probably not. If I did I would have some really deep theorems. Perhaps I will do it in the future.

Mansour: In a very recent short article[18], published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?", concluded that "Mathematical truths exist and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

Katona: I am strongly on the side of the "discovery". I think, the mathematical reality exsts without the human observer, we only find the theorems. However, it is similar to the situation where a new nice hilly area is discovered and the discoverer shows the pictures to the public. These pictures reflect the taste of the person. The same mathematics can be "discovered" in slightly different ways.

Mansour: In one of your papers[19] the two coauthors have the same family name as you. Who are they?

Katona: Gyula Y. Katona is my older son. He is a professor of mathematics at the Technical University in Budapest, working in Combinatorics. Zsolt Katona is my younger son, a professor of economics at the Berkeley University, but he has a Ph.D. also in mathematics.

Mansour: Professor Katona, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Katona: Thank you for giving me this extraordinary possibility to speak about my life and work.

[17] G. O. H. Katona, *Intersection theorems for systems of finite sets*, Acta Math. Acad. Sci. Hungar. 15 (1964), 329–337.

[18] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.

[19] G. O. H. Katona, G. Y. Katona, and Z. Katona, *Most Probably Intersecting Families of Subsets*, Combin. Prob. Comput. 21(1-2) (2012), 219–227.