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Interview with Brendan McKay

Toufik Mansour

Brendan McKay completed his Ph.D. at Melbourne University in 1980 under the supervision of Derek Allan Holton. His research interests include probability theory, combinatorics and discrete mathematics, mathematical software, the analysis of algorithms and complexity. His awards include the Australian Mathematical Society Medal (1990), Fellowship of the Australian Academy of Science (1997), Fellowship of the Australian Mathematical Society (2000), and the Combinatorial Mathematics Society of Australasia Medal for outstanding lifelong contribution (2014). He was an invited speaker at the International Congress of Mathematicians in 2010. He is serving as a member of the editorial boards of Combinatorics, Probability and Computing, Algebraic Combinatorics, the Australasian Journal of Combinatorics, Electronic Journal of Graph Theory and Applications, and MATCH Communications in Mathematical and Computer Chemistry. He has been an Editor-in-Chief of the Electronic Journal of Combinatorics since 1996. He is an Emeritus Professor in the School of Computing at the Australian National University.

Mansour: Professor Mckay, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

McKay: Combinatorics is the study of mathematical structures consisting of discrete objects and relationships between them. Except in all the cases when it is not.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

McKay: Combinatorics is little by little becoming a field of study with a broad underlying theory. The stereotype that combinatorics is just a collection of problems and results has some truth to it, but the truth is fading and acceptance is growing.

Mansour: What have been some of the main goals of your research?

McKay: Ever since my student days, my time has been divided between the computer and the blackboard. While I am at the blackboard (alas, now a whiteboard) my computer is always working on something. So, one of my passions is the use of the computer for proving theorems, not so much in the formal sense of "theorem proving", but in using the computer to apply lemmas to particular problems so that the result is a theorem. Graph generation is an essential tool, and so it remains an important component of my research.

A second passion is enumeration. In the 1980s I gave the first applications of the switching method[1] to count graphs and estimate subgraph probabilities. Later, in collaboration with Nick Wormald[2], I found how to obtain enumeration results using complex analysis in many dimensions. Both of those interests continue to be expanded.

Mansour: We would like to ask you about

Toufik Mansour is a professor of mathematics at the University of Haifa, Israel. His email address is tmansour@univ.haifa.ac.il
[1]B. D. McKay, *Subgraphs of random graphs with specified degrees*, Congressus Numerantium, 33 (1981), 213–223.
[2]B. D. McKay and N. C. Wormald, *Asymptotic enumeration by degree sequence of graphs of high degree*, European J. Combin. 11 (1990), 565–580.

your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

McKay: During my school days I had a variety of scientific interests, such as physics and chemistry, that were supported by some remarkable teachers. But I always came back to mathematics. In those years, students in high school had to write formal proofs in Euclidean geometry; I enjoyed that enormously and was easily the best in my class. So when I entered university it was natural for me to choose mathematics.

Mansour: Were there specific problems that made you first interested in combinatorics?

McKay: Mathematics students in Australia do a 6-month research project in their 4th year, which was 1975 for me. I was supposed to work on functional analysis, but it was more interesting to talk about graph theory with fellow student Chris Godsil (whose official project was in group theory). Our first interest was graph eigenvalues, starting with the problem (still unsolved!) of whether most graphs are determined by their spectrum. In that year we wrote two papers[3, 4] on eigenvalues together. In parallel, I wrote a functional analysis thesis that was so poor that some in the department wanted to block me from continuing to graduate study. But apparently, I had a champion, as I was allowed to enrol in Masters, and later in Ph.D., doing graph theory.

Mansour: What was the reason you chose Melbourne University for your Ph.D. and your advisor Derek Holton?

McKay: I grew up in Melbourne and my three sisters and I all went to Melbourne University. Derek was the only graph theorist in the department, so I was lucky to find that he was an excellent supervisor. Leaving Melbourne was an option not very energetically pursued. Probably the woman who is still my wife had something to do with that.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

McKay: I tend to be problem-driven, and in combinatorics "techniques" are more important than "theory". That is not entirely true, of course, and there have been plenty of times when I wished my theoretical knowledge was broader.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

McKay: Yes and no. When the problem is to find something (for example, to determine a probability, to characterize a class of graphs), I enjoy watching the answer appear as the calculation proceeds and do not mind if it is different from what I had guessed. Sometimes, I take advice from the computer in advance of trying to prove anything, which can save wasting time on impossible quests or suggest proof directions.

Mansour: What are the top three open questions in your list?

McKay: One of them is to understand the growth of Ramsey numbers, or even to understand why the problem is so difficult. After so much effort, why are the best bounds still exponentially far apart? More concretely, I fantasize about determining R(5,5). Vigleik Angeltveit[5] and I brought the bounds down to 43–48 and we will soon publish a further reduction to 43–46.

My work on the graph isomorphism problem[6,7] has been mostly on the practical side, but the question of the theoretical complexity has always bugged me. The recent advances of Babai[8], Grohe and Schweitzer[9] are spectacular; is that the direction in which a solution will be found?

My third example is not a single problem but more generic. In the past few years, es-

[3] C. D. Godsil and B. D. McKay, Some computational results on the spectra of graphs, Combinatorial Mathematics IV, Lecture Notes in Mathematics, 560 (Springer-Verlag, Berlin, 1976) 73-92.

[4] C. D. Godsil and B. D. McKay, *Products of graphs and their spectra*, Combinatorial Mathematics IV, Lecture Notes in Mathematics, 560 (Springer-Verlag, Berlin, 1976), 61–72.

[5] V. Angeltveit and B. D. McKay, $R(5,5) \leq 48$, J. Graph Theory 89 (2018), 5–13.

[6] B. D. McKay, *Practical graph isomorphism*, 10th. Manitoba Conference on Numerical Mathematics and Computing (Winnipeg, 1980); Congressus Numerantium 30 (1981), 45–87.

[7] B. D. McKay and A. Piperno, *Practical Graph Isomorphism, II*, J. Symbolic Comput. 60 (2014), 94–112.

[8] L. Babai, *Graph isomorphism in quasipolynomial time*, In Proceedings of the 48th Annual ACM Symposium on Theory of Computing (STOC'16) (2016), 684–697.

[9] M. Grohe, D. Neuen, and P. Schweitzer, *A faster isomorphism test for graphs of small degree*, In Proceedings of the 59th Annual IEEE Symposium on Foundations of Computer Science (FOCS) (2018), 89–100.

pecially since I started working with Misha Isaev[10], we have found general forms (such as complex martingales) of some previously ad-hoc techniques for asymptotic enumeration. However, there is still a lot of work required to apply the techniques to each application. This is unsatisfactory and I feel that there are more "master theorems" to discover.

Mansour: What kind of mathematics would you like to see in the next ten-to-twenty years as the continuation of your work?

McKay: I'm going to be bold and answer a different question. We are approaching a new era when computers will do mathematics more proficiently than humans. I do not mean doing calculations, or pedestrian case-by-case checking, which are already routine. I mean the type of mathematics that any mathematician would be proud to have their name on. When I suggested this in an online forum, I was pelted with digital rotten fruit. Responses included "mathematics requires creativity and computers are not creative" and "ChatGPT gets the wrong answer for simple logic problems". The answer to the first response is "decide what the difference is between creativity and behaviour you can not distinguish from creativity, because you will meet the latter quite soon". The second response is like judging the limitations of a human by the abilities of a day-old baby. As soon as these programs can design better versions of themselves (which has probably happened already), the snowball starts rolling. It could well be that in very few generations mathematics or mathematicians as we know them today will not exist.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

McKay: To give an example, I wrote my best-known graph generator *geng* for "pure" mathematical reasons, but (contrary to what I expected) most of the users are in "applied" areas. Was I doing pure or applied mathematics when I developed the theory that the generator is based on? It is hard to be excited by the question. I suspect that the real answer is that the pure/applied distinction arose from the administrative needs of mathematics departments and is not really about the mathematics. Be that as it may, it is hard to believe that mathematics would suffer if the distinction disappeared.

Mansour: Would you tell us about your interests besides mathematics?

McKay: In my student days I practised aikido (a martial art) but that was long ago. One of the things I spend a lot of time on now is Wikipedia, where I have been an administrator for more than 20 years.

Mansour: One of your main contributions is about *the graph isomorphism problem and its software implementation NAUTY*[11]. Could you provide an overview of the *nauty* algorithm and its significance in the field of graph theory? What motivated you to develop it?

McKay: I got interested in graph isomorphism as a student when Chris Godsil and I needed examples. The version in my Masters thesis (1976) was already much faster than previous programs and *nauty* held that position for several decades. Now there are competitive programs but *nauty* is still very popular, with more than 4,000 citations. Practitioners across science make particular use of the graph generators that *nauty* makes possible.

The key idea is to generate a list of labellings of a graph and select one of them as the "canonical labelling". Correctness requires that the canonically labelled graph is independent of the labelling of the input graph, while efficiency requires that the list of labellings is fast to compute and not too long. The two main tools are colour refinement, which takes a partition of the vertex set and makes it finer in a label-independent way, and automorphism pruning, in which symmetries of the graph are discovered and used to remove parts of the search without changing the answer.

Mansour: Together with Brinkmann, you developed *the Plantri program for generating classes of planar graphs*[12]. What is this program about?

[10] M. Isaev and B. D. McKay, *Complex martingales and asymptotic enumeration*, Random Struct. Algorithms 52 (2018), 617–661.

[11] See `https://users.cecs.anu.edu.au/~bdm/nauty/`.

[12] G. Brinkmann and B. D. McKay, *Fast generation of planar graphs*, MATCH Commun. Math. Comput. Chem. 58 (2007), 323–357.

McKay: In 1998 I published a technique for generating graphs[13] and other objects without isomorphs. Using only a small amount of memory, unique representatives of the isomorphism classes come out without the need to ever compare two graphs to each other. Plantri is an example of the method as applied to various classes of graphs embedded in the sphere. It runs very fast—for example, we made all 107,854,282,197,058 polytopal graphs on 18 vertices for one project.

Mansour: How do graphs interact with quantum systems? Would you give a few more lines about one of your publications[14] *Graph approach to quantum systems* you coauthored with Pavičić, Megill, and Fresl?

McKay: I like this question because the answer is "I do not know". One way to study quantum logic uses a class of hypergraphs that satisfy some axioms which come from the physics. These three physicists contacted me because they wanted to examine billions of hypergraphs and I know how to generate things. So I wrote a generator for their hypergraphs and we got a few joint papers. I understand physics very incompletely, and they feel the same way about my algorithms, but once we got past the barrier of speaking different mathematical languages the collaboration was successful. And great fun.

Mansour: Several of your works include some applications of graphs in chemistry. Could you explain the significance of these results in a broader context of your work?

McKay: Chemists, and especially biochemists, are big consumers of graph generation as it lets them search for interesting molecular structures. I recently published a new program for that purpose, with the help of chemist Christoph Steinbeck and his student Aziz Yirik[15]. It is free and is 100 times as fast as the premier commercial code. An unexpected side-effect is that I'm getting an amazing number of fake conference invitations.

Mansour: In one of your papers *On Ryser's Conjecture for Linear Intersecting Multipartite Hypergraphs*, coauthored with Francetić, Herke, and Wanless[16], you proved Ryser's conjecture for $r \le 9$ in the special case of linear intersecting hypergraphs. Would you tell us more about this work?

McKay: Consider a geometry of points and lines such that the points are divided into r classes and each line consists of one point from each class. In our work, we considered geometries such that each pair of lines have at least one common point. A special case of a 1967 conjecture of Ryser is that there must be a set of $r-1$ points such that every line has at least one of them. Tuza[17] proved the conjecture for $r \le 5$ in 1983. In this work, we prove it for $r \le 9$, but the general case remains open.

Mansour: In a very recent publication *Factorisation of the complete graph into spanning regular factors*, coauthored with Hasheminezhad[18], you enumerated factorizations of the complete graph into spanning regular graphs in several cases. Would you tell us more about this work?

McKay: About three decades ago, Nick Wormald and I[19,20] noticed that the asymptotic number of d-regular graphs could be written in the same form in both the very sparse and very dense regimes and conjectured that the same formula would hold in the unsolved intermediate range. The conjecture was recently proved by Nick Wormald and Anita Liebenau[21]. Since a d-regular graph can be thought of as a partition of a complete graph into d-regular and $(n-1-d)$-regular parts, the possibility of partitioning into more than two regular parts arises. I conjectured a simple

[13] B. D. McKay, *Isomorph-free exhaustive generation*, J. Algorithms 26 (1998), 306–324.

[14] M. Pavičić, B. D. McKay, N. D. Megill, and K. Fresl, *Graph approach to quantum systems*, J. Math. Physics 51 (2010), 102103.

[15] B. D. McKay, C. Steinbeck, and M. A. Yirik, *Surge - a fast open-source chemical graph generator*, J. Cheminformatics 13 (2022), #24.

[16] N. Francetić, S. Herke, B. D. McKay, and I. M Wanless, *On Ryser's conjecture for linear intersecting multipartite hypergraphs*, Europ. J. Combin. 61 (2017), 91–105.

[17] Z. Tuza, *Ryser's conjecture on transversals of r-partite hypergraphs*, Ars Combin. 16 (1983), 201–209.

[18] M. Hasheminezhad and B. D. McKay, *Factorisation of the complete graph into spanning regular factors*, Adv. Appl. Math. 146 (2023), 102487.

[19] B. D. McKay and N. C. Wormald, *Asymptotic enumeration by degree sequence of graphs of high degree*, European J. Combin 11 (1990), 565–580.

[20] B. D. McKay and N. C. Wormald, *Asymptotic enumeration by degree sequence of graphs with degrees $o(\sqrt{n})$*, Combinatorica 11 (1991), 369–382.

[21] A. Liebenau and N. C. Wormald, *Asymptotic enumeration of graphs by degree sequence, and the degree sequence of a random graph*, J. Europ. Math Soc. 26 (2024), 1–40.

variation on the two-part formula and in this paper, we proved several cases of it. More cases will appear in an upcoming paper with Misha Isaev, Angus Southwell, and Maksim Zhukovskii.

Mansour: You gave talks at numerous conferences, workshops, and seminars. What do you think about the importance of such activities for researchers?

McKay: Definitely the greatest importance is in meeting other mathematicians, sometimes engendering collaborations and other times just listening to their ideas as a way of broadening my knowledge of techniques.

Mansour: Professor McKay: *What are the Bible Codes?*

McKay: In 1990, Ilya Rips and two non-mathematicians[22] published a paper in a statistics journal claiming that information about medieval rabbis is encoded in the Hebrew Bible. On the surface, the evidence appeared very strong, but somewhat rashly I thought it would be simple to debunk. However, it was only after a very considerable effort in collaboration with others (mainly Dror Bar-Natan, Maya Bar-Hillel, and Gil Kalai[23]) that we published a paper in the same journal demolishing the claim. Of course, we didn't expect to convince the True Believers and "research" into these non-existent codes continues. This work took more time than any other paper I have written and I wouldn't have attempted it if I had known that in advance. On the one hand, scientists should not be too eager to discount exceptional claims when the stakes are very high, but on the other hand spending time on claims that have negligible chance of being true takes time away from more productive research. I was sad to hear that Rips died recently; he was a very nice person whose infatuation with the codes deprived us of a lot of first-class mathematics.

Mansour: As an advisor, you have influenced the careers of many students. What advice do you have for young mathematicians, who are just starting their academic journeys?

McKay: One piece of advice is this: If your graduate research produced a very good original idea, milk it for all it's worth. Great ideas don't come very often, so even though you need to gradually diversify your interests away from your thesis topic make sure to publish all the papers that your great idea deserves.

A second piece of advice is this: Talk to mathematicians in fields other than your own, go to their seminars, and so on. You will be surprised how often the things you have learned can be applied in places you didn't suspect. In the same spirit, search online for papers in entirely different fields of science that employ the same combinatorial objects you have been studying. Some valuable cross-discipline collaborations await you.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

McKay: I will mention one of the first enumeration problems I worked on. Approximately how many regular graphs have n vertices and degree d? In general, d is a function of n and the problem becomes harder when d grows quickly. Early results on this problem were obtained by Ron Read, Ed Bender, Rod Canfield, Nick Wormald, and Béla Bollobás[24,25]. The best result was that of Bollobás, who achieved $d \leq \sqrt{2\log n} - 1$ using the "configuration model". That method requires the estimation of a probability that tends to 0 very quickly as d increases. Bender, Canfield, and Bollobás wrote this probability as an alternating sum and estimated the terms. The difficulty is that the sum is similar to the Taylor expansion of e^{-d^2}: as d increases, the value of the sum shrinks while the sizes of the largest terms grow. Very soon, the terms cannot be estimated accurately enough to give a useful value for the sum. My "Eureka moment" was to guess that the reciprocal of the probability should satisfy a sum similar to the Taylor expansion of e^{+d^2}, which has positive terms. And, indeed, such a sum exists and its terms

[22] See `https://en.wikipedia.org/wiki/Eliyahu_Rips`.

[23] B. D. McKay, D. Bar-Natan, M. Bar-Hillel, and G. Kalai, *Solving the Bible code puzzle*, Statistical Science 14 (1999), 150–173.

[24] E. A. Bender and R. W. Canfield, *The asymptotic number of labelled graphs with given degree sequences*, J. Combin. Theory, Ser. A. 24 (1978), 296–301.

[25] B. Bollobás, *A probabilistic proof of an asymptotic formula for the number of labelled regular graphs*, European J. Combin. 1:4 (1980), 311–316.

can be estimated using the "switching method" that I had earlier used on another problem. I managed this for $d = o(n^{1/3})$. Nick Wormald[20] and I later pushed the same approach up to $d = o(n^{1/2})$, and all the remaining ranges of d have now been filled in using different methods, by myself, Anita Liebenau, and Nick Wormald.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

McKay: When I was finding the eigenvalue distribution of random regular graphs[26] in the early 1980s, one of the steps was to prove that they had few short cycles. That particular point turned out to be known already, so the method I developed did not appear until another paper where I gave bounds on the probability of a subgraph in a random graph with given degrees. The case of general subgraphs of graphs with arbitrary degree sequences is still incompletely solved 43 years later. There is not any method that works in all cases, so progress has involved solving one subcase after another, often inventing new methods. Nick Wormald, Catherine Greenhill, and Misha Isaev have contributed to various aspects and we have plans for how to fill in some large gaps.

Mansour: In a very recent short article[27], published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?", concluded that "Mathematical truths exist and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

McKay: I have always thought this argument is a bit pointless because it inevitably devolves into petty nitpicking about definitions of words. I would say that mathematical truths are discovered because they were true already before the discovery. The alternative leads to absurdity: if two mathematicians independently prove the same theorem one minute apart, did one of them invent it and the other discover it? Surely it makes more sense to say that they both did the same thing. However, while I agree with Nathanson on that point, I disagree with the conclusions he draws from it.

Mansour: Professor McKay, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

McKay: Thank you for the invitation.

[26] B. D. McKay, *The expected eigenvalue distribution of a large regular graph*, Linear Alg. Appl. 40 (1981), 203–216.

[27] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.