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Interview with Johann Cigler

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Johann Cigler completed his Ph.D. at the University of Vienna in 1960 under the supervision of Edmund Hlawka. After some years as an assistant at the Universities of Mainz and Vienna, in 1964 he got a professorship in abstract analysis at the University of Groningen and in 1970 a professorship in mathematics at the University of Vienna, where he subsequently became emeritus in 2005. In 1994 he was elected a full member of the Austrian Academy of Sciences. Apart from some introductory textbooks that resulted from his teaching activities his publications over the years dealt with uniform distribution, ergodic theory, harmonic analysis on locally compact Abelian groups, functors on categories of Banach spaces, and finally Enumerative Combinatorics.

Mansour: Professor Cigler, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Cigler: I do not consider myself a combinatorialist. Over the years, my mathematical interests have changed, but basically classical analysis has always played a dominant role. At present some methods of enumerative combinatorics are in the focus of my interest, such as generating functions, orthogonal polynomials, or Hankel determinants. I cannot give a definition of enumerative combinatorics but would describe it roughly as a part of mathematics that originated from the wish to count numbers associated with finite mental objects.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Cigler: I have the impression that combinatorics is already an established part of mathematics.

Mansour: What have been some of the main goals of your research?

Cigler: I have put most of my energy into teaching. This has of course influenced my research. Thus my main goal was the desire to really understand things and to present them in a form that emphasizes analogies with concrete facts or typical examples.

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Cigler: My parents ran a market garden for flowers and my ancestors were farmers or craftsmen. No one had more than an elementary education. I was lucky enough to be allowed to attend a Gymnasium (=high-school). So, my earliest experiences with mathematics were only the things I learned in school.

Mansour: Were there specific problems that made you first interested in combinatorics?

Cigler: No, not really. The book "Finite operator calculus" by Gian-Carlo Rota[1] seduced me to combinatorics.

Mansour: What was the reason you chose the University of Vienna for your Ph.D. and your advisor Edmund Hlawka?

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[1] G.-C. Rota, *Finite operator calculus*, With the collaboration of P. Doubilet, C. Greene, D. Kahaner, A. Odlyzko and R. Stanley, Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, 1975.

Cigler: I lived near Vienna where the University and the Technische Hochschule had mathematics departments. As I was hardly interested in technology I chose the University. The mathematics department had three professors who alternately taught the few required courses. One of them was Edmund Hlawka. He was an inspiring teacher who worked on uniform distribution modulo 1 which I also liked at that time. So I chose him as my thesis advisor.

Mansour: What would guide you in your research? A general theoretical question or a specific problem?

Cigler: Two beautiful formulae that caught my attention led me to some of my papers: The q-binomial theorem[2,3,4] in the form $(A+B)^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q A^k B^{n-k}$ for operators A, B on the polynomials which satisfy $BA = qAB$ and the formula $\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-k}{k} = F_{n+1} = \sum_{i\in\mathbb{Z}} (-1)^i \binom{n}{\lfloor \frac{n+5i}{2} \rfloor}$ for the Fibonacci numbers and its q-analog which Schur[5,6,7] has used for his proof of the Rogers-Ramanujan identities.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Cigler: Yes, if it is true for sufficiently many special cases.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Cigler: My knowledge of combinatorics is not broad enough to give a meaningful answer. I can only say what has influenced my own knowledge of combinatorics. I already mentioned "Finite operator calculus" by Gian-Carlo Rota[1], then the combinatorial theory of orthogonal polynomials by Gérard Viennot[8] and the Russian original of the book "Integral representation and the computation of combinatorial sums" by Georgiĭ Egorychev[9], which I received from Zentralblatt für Mathematik for review.

Mansour: As an advisor, you have influenced the careers of many students. What advice do you have for young mathematicians who are just starting their academic journeys?

Cigler: This is difficult to answer, but I have observed that those students who chose their thesis topics themselves from the problems I mentioned in my courses and worked on them in their own way had the most success.

Mansour: Do you think that there are core or mainstream areas in mathematics? Are some topics more important than others?

Cigler: I like to compare mathematics with a living organism where all parts are necessary for its health.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case? How do you see the relationship between so-called "pure" and "applied" mathematics?

Cigler: I think the dual role of applied and pure gives mathematics its vitality.

Mansour: Would you tell us about your interests besides mathematics?

Cigler: I always loved reading everything I could get my hands on.

Mansour: You have also worked on the combinatorics of lattice paths[10]. Could you describe some of the key results in this area and how they connect to other areas of mathematics, such as probability or statistical mechanics?

Cigler: I used lattice paths only as combinatorial tools or illustrative examples for other things such as the computation of Hankel de-

[2] G. E. Andrews, R. Askey, and R. Roy, *Special functions*, Encyclopedia of Mathematics and its applications 71.

[3] M.-P. Schützenberger, *Une interprétation de certaines solutions de l'équation fonctionelle: $F(x+y) = F(x)F(y)$*, C.R. Acad. Sci. Paris 236 (1953), 352–353

[4] J. Cigler, *Operatormethoden für q-Identitäten*, Monatshefte für Mathematik 88 (1979), 87–105

[5] G.E. Andrews, *A polynomial identity which implies the Rogers-Ramanujan identities*, Scripta Math. 28 (1970) 297–305.

[6] J. Cigler, *q-Fibonacci polynomials and the Rogers-Ramanujan identities*, Ann. Comb. 8:3 (2004), 269–285.

[7] I. Schur, *Ein Beitrag zur additiven Zahlentheorie und zur Theorie der Kettenbrüche* 1917, In: Gesammelte Abhandlungen, Bd.2, Springer-Verlag, 1973, 117–136.

[8] G. Viennot, *A combinatorial theory for general orthogonal polynomials with extensions and applications*, In: Brezinski, C., Draux, A., Magnus, A.P., Maroni, P., Ronveaux, A. (eds) Polynômes Orthogonaux et Applications, Lecture Notes in Mathematics 1171, Springer, Berlin, Heidelberg, 1985.

[9] G. P. Egorychev, *Integral representation and the computation of combinatorial sums*, Transl. Math. Monogr. 59, American Mathematical Society, Providence, RI, 1984.

[10] For example, see J. Cigler, *Fibonacci-Zahlen, Gitterpunktwege und die Identitäten von Rogers-Ramanujan, Fibonacci numbers, lattice point paths and Rogers-Ramanujan identities*, Math. Semesterber. 52:2 (2005), 97–125.

terminants[11]. I know too little about their role in other parts of mathematics.

Mansour: In your research, you have often used the concept of q-analogs. Could you explain what q-analogs are and why they are useful in combinatorics?

Cigler: Superficially q-analogs are generalizations of formulas to which they reduce for $q = 1$. In many cases you can associate a weight to some objects which depends on a parameter q and equals 1 for $q = 1$. Then any identity about the weights is a q-analog of corresponding identities for the numbers of the objects. To give a simple example, if we associate the weight q^k to a permutation with k inversions and if $I(n,k)$ denotes the number of permutations of n elements with k inversions then the formula[12]

$$\prod_{j=1}^{n}(1+q+\cdots+q^{j-1}) = \sum_{k} I(n,k)q^k$$

is a q-analog of the fact that there are $n!$ permutations of n elements since the left-hand side is a q-analog of $n!$ and the right-hand side is the weight of the permutations.

Mansour: Your work in q-series and combinatorial identities has been widely recognized. What initially drew you to these areas of mathematics?

Cigler: I am mostly thinking in analogies. Therefore, q-identities immediately attracted my attention.

Mansour: Professor Cigler, what are *q-Chebyshev polynomials*[13]?

Cigler: They are polynomials with the property that their coefficients and their recurrences are natural q-analogs of those of the classical Chebyshev polynomials $U_n(x)$ and $T_n(x)$.

For example, the bivariate polynomials $U_n = U_n(x,s,q)$ satisfy

$$U_n = (1+q^n)xU_{n-1} + q^{n-1}sU_{n-2}$$

with initial values $U_0 = 1$ and $U_1 = (1+q)x$. The bivariate polynomials $T_n = T_n(x,s,q)$ satisfy

$$T_n = (1+q^{n-1})xT_{n-1} + q^{n-1}sT_{n-2}$$

with initial values $T_0 = 1$ and $T_1 = x$.

Mansour: In your paper *Some algebraic aspects of Morse code sequences*[14], you studied algebraic and combinatorial aspects of Morse code sequences. Would you elaborate more on this work?

Cigler: A Morse code sequence is a finite sequence of dots a and dashes b. If a dot has length 1 and a dash has length 2 then the number of all sequences of length $n-1$ is the Fibonacci number F_n. We interpret the set of all Morse code sequences as a monoid with respect to concatenation and consider the monoid algebra of all finite linear combinations of words in a, b. In the general case a and b do not commute and the length of b can be any positive number. By considering various homomorphisms we get different generalizations and q-analogs of Fibonacci and Lucas polynomials.

Mansour: In one of your recent papers, a joint paper with Christian Krattenthaler *Bounded Dyck paths, bounded alternating sequences, orthogonal polynomials, and reciprocity*[15], among other results, you extended combinatorial reciprocity to families of non-intersecting bounded up-down paths and certain arrays of alternating sequences which you called *alternating tableaux*. Would you tell us about this work?

Cigler: If a_n is the number of certain combinatorial objects of size n and the sequence a_n satisfies a linear recurrence with constant coefficients then the sequence can be extended backward to negative n. If these numbers too have a combinatorial interpretation then we speak of a reciprocity law. Answering a question of mine about extending the number of bounded Dyck paths to negative n in MathOverflow Richard Stanley[16] gave a combinatorial interpretation. This interpretation together with a determinant representation for negative n turned out to be just the "peak of an iceberg" which has been "lifted out of the sea" with the help of Christian Krattenthaler.

[11] J. Cigler and C. Krattenthaler, *Hankel determinants of linear combinations of moments of orthogonal polynomials*, Int. J. Number Theory 17:2 (2021), 341–369.

[12] Muir, *On a simple term of a determinant*, Proc. Royal S. Edingburgh 21 (1898-9), 441–477.

[13] J. Cigler, *q-Chebyshev polynomials*, arXiv:1205.5383.

[14] J. Cigler, *Some algebraic aspects of Morse code sequences*, Discrete Math. Theor. Comput. Sci. 6:1 (2003), 55–68.

[15] J. Cigler and C. Krattenthaler, *Bounded Dyck paths, bounded alternating sequences, orthogonal polynomials, and reciprocity*, European J. Combin. 121 (2024), Paper No. 103840.

[16] See `https://mathoverflow.net/questions/372642/number-of-bounded-dyck-paths-with-negative-length`.

Mansour: How are *Hankel determinants* and *Catalan numbers* connected?

Cigler: The sequence of Catalan numbers is the uniquely determined sequence u_n with the property that all Hankel determinants of the sequences (u_n) and (u_{n+1}) are 1.

Mansour: You gave talks at numerous conferences, workshops, and seminars. What do you think about the importance of such activities for researchers?

Cigler: I think it is very important to be familiar with the state of the art in your field. But how that happens is not the same for everyone. It seems that for most people this happens through participation in conferences, workshops, and seminars. But for various reasons, this does not work for me. I really need written texts. Therefore, I obtained my mathematical knowledge almost exclusively from books and other writings. In the first semesters of my study these were "Theorie und Anwendung der unendlichen Reihen" by Knopp[17] and "Aufgaben und Lehrsätze aus der Analysis" by Polya and Szegö[18] and somewhat later the first Russian edition of "Normed rings" by Naĭmark,[19], which strongly influenced my view of mathematics. I often look at MathStackExchange or MathOverflow for advice and/or inspiration. An almost indispensable aid is also the Online Encyclopedia of Integer Sequences OEIS.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Cigler: Let me sketch how I arrived at the q-Chebyshev polynomials. I was very interested in Fibonacci and Lucas polynomials. Both are orthogonal with respect to some linear functional and their even moments are the Catalan numbers and the central binomial coefficients respectively. I tried to find nice orthogonal q-analogs where the moments also are nice q-analogs of the Catalan numbers and the central binomial coefficients. Carlitz[20] had already introduced q-analogs of the Fibonacci polynomials which are orthogonal but do not have nice moments and the corresponding Lucas polynomials are not even orthogonal. So I looked for other q-analogs with simple formulas and simple recurrences. After many attempts, I found special cases of the Al-Salam and Ismail[21] polynomials with the desired properties. Surprisingly their moments turned out to be related to the nice q-analogs of the Catalan numbers which Andrews[22] had introduced for other reasons. So the q-Chebyshev polynomials could be defined by their recursions as shown above or as the orthogonal polynomials whose moments are the Andrews q-Catalan numbers. The wish to better motivate these definitions led me to the following approach in which the q-analogs of Andrews appear in a natural way albeit in a different form.

Consider the monic Chebyshev polynomials of the second kind $u_n(x)$ and observe that their even moments $C_n/4^n$ can also be written as $2(2n-1)!!/(2n+2)!!$. This formula suggests $(1+q)[2n-1]_q!!/[2n+2]_q!!$ as natural q-analogs. These turn out to be essentially the same as the q-analogs of Andrews and are the even moments of nice orthogonal polynomials which are the monic versions of the desired q-Chebyshev polynomials of the second kind.

The monic Chebyshev polynomials of the first kind $t_n(x)$ have moments $(2n-1)!!/(2n)!!$. This leads in the same way to the monic versions of q-Chebyshev polynomials of the first kind.

From the first to the last steps, it took approximately 15 years.

Mansour: In a very recent short article[23], published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?", concluded that "Mathematical truths exist and mathematicians only discover them."

[17] K. Knopp, *Theorie und Anwendung der unendlichen Reihen*, Springer-Verlag, Berlin-Heidelberg, 1947.

[18] G. Pólya and G. Szegö, *Aufgaben und Lehrsätze aus der Analysis*, Dover Publications, New York, 1945.

[19] M. A. Naĭmark, *Normed rings*, Wolters-Noordhoff Publishing, Groningen, 1970.

[20] L. Carlitz, *q-Fibonacci polynomials*, Fib. Quart.13 (1975), 97–102.

[21] W. A. Al-Salam and M. E. H. Ismail, *Orthogonal polynomials associated with the Rogers-Ramanujan continued fraction*, Pacific J. Math. 104 (1983), 269–283.

[22] G. E. Andrews, *Catalan numbers, q-Catalan numbers and hypergeometric series*, J. Comb. Theory Ser. A 44:2 (1987), 267–273.

[23] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.

On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

Cigler: I have the feeling that some definitions are invented and that other things are discovered. Otherwise, it would not be understandable why things are so often found independently of each other. For example, the q-Chebyshev polynomials have also been found by Atakishiyeva and Atakishiyev[24] as special cases of big q-Jacobi polynomials and by Marberg and White[25] in a totally different context.

Mansour: Professor Cigler, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Cigler: Professor Mansour, I thank you very much for the interviews which I have read with great interest. I found it very enlightening to learn about the motivations, goals, and different kinds of thinking of other mathematicians which you hardly get told otherwise.

[24] M. Atakishiyeva and N. Atakishiyev, *On discrete q-extensions of Chebyshev polynomials*, Commun. Math. Anal. 14 (2013), 1–12.

[25] E. Marberg and G. White, *Variations of the Poincare series for affine Weyl groups and q-analogs of Chebyshev polynomials*, Adv. Appl. Math. 82 (2017), 129–154.