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Interview with Isabella Novik

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Photo by Lev Novik

Isabella Novik received her Ph.D. from the Hebrew University of Jerusalem in 1999 under the supervision of Gil Kalai. She was a Morrey Assistant Professor at the University of California, Berkeley, and then an Acting Assistant Professor at the University of Washington (2001—2004). She has remained at the University of Washington ever since and is now a Professor of Mathematics there. Her research focuses on algebraic, geometric, and topological combinatorics, with particular emphasis on the combinatorics of polytopes and manifolds and their connections to commutative algebra and algebraic topology. Her honors include the Haim Nessyahu Prize in Mathematics, an Alfred P. Sloan Research Fellowship, and election as a Fellow of the American Mathematical Society. She also serves on the editorial boards of *Algebraic Combinatorics*, *Proceedings of the American Mathematical Society*, *SIAM Journal on Discrete Mathematics*, and on the Advisory board of the new journal *Combinatorial Commutative Algebra*.

Mansour: Professor Novik, first of all, we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Novik: Combinatorics is the study of finite structures and their enumeration. Such structures appear in essentially every area of mathematics: algebra, for instance through finite groups or finitely generated rings and modules; geometry and topology, for example via triangulations of nice topological spaces; analysis, such as approximating convex bodies by polytopes; and many others. A friend once joked that at his university, every talk that does not fit neatly into a "standard" area is classified as a combinatorics talk. I rather like that point of view.

Mansour: What do you think about the development of the relations between combinatorics and the rest of mathematics?

Novik: I think one of the best things about combinatorics is the way it interacts with the rest of mathematics. Many other areas frequently rely on combinatorial tools and results, but the relationship goes both ways: when working in combinatorics, we constantly draw on techniques, results, and insights from other fields. I find these connections especially rewarding.

Mansour: What have been some of the main goals of your research?

Novik: I try to understand the face numbers of triangulations of nice topological spaces, such as manifolds or normal pseudomanifolds[1,2,3]. These face numbers are usually recorded in a single vector, called the f-vector. A central question is how the topology of a given space constrains the possible f-vectors of its triangu-

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[2] I. Novik and E. Swartz, *Applications of Klee's Dehn–Sommerville relations*, Discrete Comput. Geom. 42:2 (2009), 261–276.
[3] S. Murai and I. Novik, *Face numbers and the fundamental group*, Israel J. Math. 222:1 (2017), 297–315.

lations.

I have also been working to understand face numbers of triangulated spheres—and more generally, manifolds and pseudomanifolds—that carry additional structure, such as central symmetry[4], meaning that the complex possesses a free involution; balancedness, meaning that the vertices of a $(d-1)$-dimensional complex can be colored with d colors so that no edge is monochromatic[5,6]; or flagness, meaning that every clique in the graph forms a face. These constraints produce beautiful families of objects whose combinatorial behavior is still poorly understood[4,7].

Mansour: We would like to ask you about your formative years. What were your early experiences with mathematics? Did that happen under the influence of your family or some other people?

Novik: My mom worked as a school math teacher and my dad as a theoretical physicist, so yes—I was surrounded by mathematics from the very beginning. In fact, it was my mom who first took me to a math olympiad, which eventually led me to join a math circle. During my first year in the math circle, I'm not sure who was more excited about the weekly problem sets—me or my dad. The math circle itself was, I think, the most influential experience of all.

Mansour: Were there specific problems, results, or examples that first made you interested in combinatorics?

Novik: Many math olympiad problems are essentially baby results in combinatorics—I was always intrigued by how much you can sometimes derive from really simple tools, like the pigeonhole principle or coloring arguments. (For example: can we tile a 10×10 board with 1×4 rectangles?) Early on, though, I especially enjoyed problems in elementary number theory, such as Fermat's little theorem or Wilson's theorem[8]. Of course, these results can also be viewed as part of combinatorics.

Mansour: What was the reason you chose the Hebrew University of Jerusalem for your Ph.D. and Gil Kalai[9] as your advisor?

Novik: I was an undergraduate at the Hebrew University and started working with Gil Kalai in my second year. Gil was the instructor in the advanced calculus class I took, so when I knocked on his door one day and asked whether we could read something together, his first reaction was to point out that his research wasn't in analysis. In any case, we started working together, and that was that—I never considered leaving.

Mansour: What would guide you in your research more often: a general theoretical question or a specific problem?

Novik: I am usually guided by specific problems. And thanks in large part to Gil, the field has no shortage of conjectures and open questions.

Mansour: When you are working on a problem, do you feel that something is true even before you have the proof?

Novik: Yes—this happens quite often. Many conjectural statements are simply too beautiful to imagine they could be false. Of course, every now and then they do turn out to be false, and that is usually very disappointing.

Mansour: What three results do you consider the most influential in combinatorics during the last thirty years?

Novik: So much progress has happened over the last thirty years that it is genuinely hard to pick only three results. Let me focus on results that are closer to my area of expertise. One is Paco Santos' disproof of the Hirsch conjecture[10] on the diameter of a d-polytope with n facets. Another is Adiprasito's[11] proof of

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[5] M. Juhnke-Kubitzke, S. Murai, I. Novik, and C. Sawaske, *A generalized lower bound theorem for balanced manifolds*, Math. Z. 289 (2018), no. 3-4, 921–942.

[6] S. Klee and I. Novik, *Lower bound theorems and a generalized lower bound conjecture for balanced simplicial complexes*, Mathematika 62:2 (2016), 441–477.

[7] I. Novik and H. Zheng, *Lower bounds on the g-numbers of spheres without large missing faces*, arXiv:2604.16905.

[8] P. G. Anderson, A. T. Benjamin, and J. A. Rouse, *Combinatorial Proofs of Fermat's, Lucas's, and Wilson's Theorems*, Amer. Math. Monthly 112:3 (2005), 266–268.

[9] *Interview with Gil Kalai*, Enumer. Combin. Appl. 2:2 (2022), Interview #S3I7.

[10] F. Santos, *A counterexample to the Hirsch conjecture*, Ann. of Math. (2) 176:1 (2012), 383–412.

[11] K. Adiprasito, *Combinatorial Lefschetz theorems beyond positivity*, arXiv:1812.10454.

[12] S. A. Papadakis and V. Petrotou, *The characteristic 2 anisotropicity of simplicial spheres*, arXiv:2012.09815.

[13] K. Adiprasito, S. A. Papadakis, and V. Petrotou, *Anisotropy, biased pairings, and the Lefschetz property for pseudomanifolds and cycles*, arXiv:2101.07245.

the g-conjecture for spheres, which was subsequently simplified by Papadakis–Petrotou[12], Adiprasito–Papadakis–Petrotou[13], and Karu–Xiao[14]. A third is the disproof of the topological Tverberg conjecture by Blagojević, Frick, and Ziegler[15,16], building on work of Mabillard and Wagner[17,18].

Mansour: What are the top three open questions in your list?

Novik: One such question is Gal's conjecture[19] on the face numbers of flag simplicial spheres. More generally, Eran Nevo[20] proposed conjectures on the face numbers of simplicial $(d-1)$-spheres whose missing faces all have dimension at most k. As k ranges from 1 to d, these conjectures interpolate between Gal's conjecture for flag spheres and the generalized lower bound theorem for all simplicial spheres.

A second question concerns understanding the face numbers of centrally symmetric (cs) simplicial spheres and cs simplicial polytopes in terms of the dimension and the number of vertices[4,21]. For cs spheres, a tight upper bound theorem is known, but for cs polytopes there is not even a plausible upper bound conjecture. On the other hand, although Stanley[22,23] proved a natural cs analogue of the generalized lower bound theorem for cs polytopes, whether the same result holds for cs simplicial spheres remains wide open.

The third question is to characterize the face numbers of simplicial balls. Billera and Lee[24] showed that the g-theorem for spheres implies certain necessary conditions on the face numbers of balls, and they conjectured that these conditions are also sufficient. Kolins[25] later disproved this conjecture. At present, we do not even have a plausible replacement conjecture.

Mansour: What kind of mathematics would you like to see in the next ten to twenty years as the continuation of your work?

Novik: Any progress in face enumeration would make me happy.

Mansour: What do you think about the distinction between pure and applied mathematics that some people focus on? Is it meaningful at all in your case?

Novik: I know very little about applied mathematics, so I cannot give a meaningful answer. At the moment, my research is firmly on the pure side. That said, with the rapid development of AI and computational tools, it is hard to predict how everything will change in the next few years, so I may well have a very different answer in the future.

Mansour: Your work connects combinatorics with areas like commutative algebra and topology. Was there a moment when you realized these connections were not just technical tools, but actually changed the way you think about mathematics?

Novik: In combinatorics, we count things, but numbers by themselves do not carry much meaning. On the other hand, when the numbers we study arise as invariants of mathematical objects—such as dimensions of vector spaces—and when there are natural maps between these objects, the numbers and the relationships between them, for example various inequalities, suddenly become much more transparent and meaningful. This is why commutative algebra and topology are so useful: they provide additional structure and interpretation for the objects we study. In that sense, the connections to these areas are not merely technical tools but natural extensions of the underlying combinatorial problems.

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[15]F. Frick, *Counterexamples to the topological Tverberg conjecture*, Oberwolfach Rep. 12:1 (2015), 318–321.
[16]P. V. M. Blagojević, F. Frick, and G. M. Ziegler, *Barycenters of polytope skeleta and counterexamples to the topological Tverberg conjecture, via constraints*, J. Eur. Math. Soc. 21:7 (2019), 2107–2116.
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[21]I. Novik, *Face numbers: the upper bound side of the story*, ICM—International Congress of Mathematicians. Vol. 6. Sections 12-14, 4622–4644, EMS Press, Berlin, 2023.
[22]R. P. Stanley, *On the number of faces of centrally-symmetric simplicial polytopes*, Graphs Combin. 3:1 (1987), 55–66.
[23]*Interview with Richard P. Stanley*, ECA 1:1 (2021), Interview #S3I1.
[24]L. J. Billera and C. W. Lee, *A proof of the sufficiency of McMullen's conditions for f-vectors of simplicial convex polytopes*, J. Combin. Theory 31 (1981), 237–255.
[25]S. Kolins, *f-vectors of triangulated balls*, Discrete Comput. Geom. 46:3 (2011), 427–446.

Mansour: Combinatorics is often seen as very discrete and concrete, yet your work reveals hidden geometric or topological structure. Do you feel like you are discovering these structures, or inventing the right language to see them?

Novik: I am not sure I am doing either of those things. Very often I work on face numbers of triangulations of nice topological objects or on face numbers of polytopes, and so I am mostly just trying to use the structure that is naturally there.

Mansour: In your ICM talk[21] *Face numbers: the upper bound side of the story*, you revisit upper bound phenomena through the language of neighborliness, polytopes, spheres, and manifolds. Looking back at this line of work, which ideas or methods do you feel most profoundly changed our understanding of extremal face numbers, and which open problem in this direction do you find the most compelling today?

Novik: This is a fascinating part of face enumeration. Over the years, there have been many crucial ideas and developments, and my answer can only touch on a few highlights.

The story begins with cyclic polytopes, discovered and rediscovered by Carathéodory[26], Motzkin[27], Gale[28], and others. These polytopes have remarkable properties: for example, the $\lfloor d/2 \rfloor$-skeleton of a d-dimensional cyclic polytope with n vertices coincides with the $\lfloor d/2 \rfloor$-skeleton of an $(n-1)$-simplex. In particular, the graph of a 4-dimensional cyclic polytope with n vertices is the complete graph K_n.

A major conceptual advance was McMullen's introduction[29] of the h-numbers, obtained from the f-numbers by an invertible linear transformation. These numbers have combinatorial, algebraic, and geometric interpretations, and are far more convenient to work with than the face numbers themselves. For more than fifty years, the h-numbers and their consecutive differences—the g-numbers—have been the central invariants in face-enumeration theory.

Another major idea is that the face numbers of simplicial polytopes, simplicial spheres—that is, triangulations of spheres—and more generally Eulerian simplicial complexes, introduced by Vic Klee[30] in 1964, satisfy certain linear relations known as the Dehn–Sommerville relations[31]. In the language of the h-numbers, these relations say that the h-vector of an Eulerian complex is symmetric.

In my view, the most profound development was the introduction of Stanley–Reisner rings[32,33,34], also known as face rings. This brought powerful tools from commutative algebra and algebraic geometry into the subject and fundamentally changed how we study face numbers. As an example, Hochster's formula[35] relates the local cohomology of the Stanley–Reisner ring of a complex K to the Betti numbers, certain topological invariants, of specific subcomplexes of K.

I should also mention Shemer's work[36] on neighborly polytopes, which introduced new techniques for constructing many neighborly polytopes. This, in turn, inspired the search for even more neighborly polytopes and neighborly spheres.

There are still many compelling open problems. One is Gil Kalai's conjecture[37] that neighborly spheres are much more common than we currently know how to construct. Another is Klee's conjecture[38] that the Upper Bound Theorem should extend to all Eulerian simplicial complexes. A further challenge is to find tight upper bounds for centrally symmetric polytopes.

Mansour: Much of your work focuses on centrally symmetric polytopes and spheres. From

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[37] G. Kalai, *Many triangulated spheres*, Discrete Comput. Geom. 3 (1988), 1–14.
[38] V. Klee, *On the number of vertices of a convex polytope*, Canad. J. Math. 16 (1964), 701–720.

your perspective, what role does central symmetry play in problems about face numbers? Does it mainly impose constraints, or does it lead to new kinds of extremal structures?

Novik: Central symmetry fascinates me for the following reason. Although the boundary complexes of simplicial polytopes form only a tiny fraction of all simplicial spheres, they share the same collection of f-vectors—this is a consequence of the celebrated g-theorem[39,11–14]. However, once we restrict to centrally symmetric (cs) objects, the situation changes drastically: the set of f-vectors of cs simplicial polytopes is very different from the set of f-vectors of cs simplicial spheres.

One illustration of this difference comes from the natural analogue of neighborliness for cs complexes, called cs-neighborliness. There exist cs simplicial $(d-1)$-spheres with arbitrarily many vertices that are cs-$\lfloor d/2 \rfloor$-neighborly[40]. In contrast, maximal cs-neighborliness for cs polytopes is quite restrictive: a cs d-polytope with more than 2^d vertices cannot even be cs-2-neighborly[41].

This connects to an open problem I mentioned earlier: finding tight upper bounds on the face numbers of cs polytopes. At present, we have no idea what the maximizing cs polytopes should look like. We do not even know whether, for sufficiently large d and n, there exists a cs d-polytope with $2n$ vertices that simultaneously maximizes all face numbers.

Mansour: In your recent work with Hailun Zheng[42], *Transversal numbers of simplicial polytopes, spheres, and pure complexes*, you introduce a family of d-dimensional polytopes that can be viewed as "siblings" of cyclic polytopes, and you obtain new lower bounds for transversal ratios. Could you explain the intuition behind this construction? Do you expect transversal extremal theory to develop into a framework parallel to the classical extremal theory of face numbers?

Novik: The construction is designed to mimic the Gale evenness condition. This theorem characterizes the facets of $C(d,n)$, the cyclic d-polytope with vertex set $[n] := \{1, 2, \dots, n\}$. In particular, it implies that each $2k$-subset of $[n]$ of the form

$$\{i_1 < i_1 + 1 < i_2 < i_2 + 1 < \cdots < i_k < i_k + 1\}$$

is a facet of $C(2k,n)$. Moreover, the simplicial complex generated by these facets, $B(2k,n)$, is a $(2k-1)$-ball whose boundary is isomorphic to the boundary complex of the cyclic polytope $C(2k-1,n)$.

Each set of the above form is a disjoint union of k consecutive pairs. It is then quite natural to generalize this idea and instead look at subsets of $[n]$ that are a disjoint union of $k-1$ consecutive pairs followed by a consecutive triple, or a disjoint union of $k-1$ consecutive pairs followed by a consecutive quadruple. In the same spirit, the simplicial complexes $D(2k+1,n)$ and $D(2k+2,n)$, whose facets are given by these two collections, turn out to be simplicial $2k$- and $(2k+1)$-balls, respectively. The boundaries of these balls are k-neighborly $(2k-1)$- and $2k$-spheres. Perhaps more surprisingly, these spheres are realizable as the boundary complexes of polytopes, which we call "siblings" of cyclic polytopes.

The relation to transversal ratios is as follows. Recall that the transversal ratio of a pure simplicial complex Δ is the minimum proportion of vertices required to intersect every facet of Δ. It is not hard to see from the Gale evenness condition that the transversal ratio of $C(2k,n)$ approaches $1/2$ as $n \to \infty$. On the other hand, each facet of $C(2k-1,n)$ contains either 1 or n, so the transversal ratio of $C(2k-1,n)$ is $2/n$, which is very small. By contrast, the transversal ratios of the two families of spheres described above approach $1/2$ and $2/5$, respectively.

Recently, there was a striking development on transversal numbers: Michael Dobbins and Seunghun Lee[43] proved that for every $d \geq 5$, there exists a family of simplicial polytopes whose transversal ratio approaches 1 as the number of vertices grows. It remains open whether the transversal ratios of simplicial 4-polytopes are bounded away from 1. Another major open problem is, for $d \geq 5$, understanding how transversal ratios of spheres behave

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[40] I. Novik and H. Zheng, *Highly neighborly centrally symmetric spheres*, Adv. Math. 370 (2020), 107238.
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[42] I. Novik and H. Zheng, *Transversal numbers of simplicial polytopes, spheres, and pure complexes*, SIAM J. Discrete Math. 40:1 (2026), 192–213.
[43] M. G. Dobbins and S. Lee, *Polytopes with large transversal ratio*, arXiv:2603.16298.

asymptotically, that is, how quickly they approach 1. These and many additional problems will hopefully keep the field active and lead to the construction of numerous new and interesting polytopes and spheres. The Dobbins–Lee result suggests that transversal extremal theory may be richer than we currently understand.

Mansour: In the paper *Affine stresses, inverse systems, and reconstruction problems*[44], you prove Kalai's reconstruction conjecture for all $d \geq 5$ and establish connections between affine stresses, inverse systems, and lower bound phenomena. How did affine stresses become such a central tool in your work? What aspects of reconstruction from stress spaces do you feel are still not fully understood?

Novik: Let me first explain what these objects are. An affine k-stress on a simplicial complex Δ, whose vertices are embedded in $\mathbb{R}^d$, is an assignment of weights to the $(k-1)$-faces of Δ such that certain "equilibrium" conditions are satisfied at every $(k-2)$-face. The k-stresses form a vector space. In the case $k = 1$, this space coincides with the space of affine dependences among the vertices, so affine stresses may be viewed as a higher-dimensional analogue of affine dependence. When $k = 2$, the weights are assigned to edges, and we recover the classical objects from the rigidity theory of frameworks.

Like with the Stanley–Reisner rings, affine stresses form an important tool for studying face numbers. This line of work was pioneered by Gil Kalai[45] in the late 1980s. He observed that if $d \geq 4$, then the graph of a simplicial $(d-1)$-manifold Δ is generically d-rigid, and hence the dimension of the space of affine 2-stresses, under a generic embedding in $\mathbb{R}^d$, is given by $g_2(\Delta) := h_2(\Delta) - h_1(\Delta)$. This immediately implies that $g_2(\Delta)$ is nonnegative, and it also allows one to characterize the "minimizers," that is, manifolds with $g_2 = 0$.

The theory of higher-dimensional affine stresses was developed in the 1990s by Lee[46], and also by Tay, White, and Whiteley[47,48]. The spaces of these stresses are related to Macaulay inverse systems of certain ideals containing the Stanley–Reisner ideal. Studying affine stresses often leads to new lower-bound-type inequalities on face numbers, and this is really how affine stresses became one of the central tools in my work.

As for the reconstruction conjectures, Gil actually proposed two of them. The first concerns reconstructing the *combinatorial* type of a d-polytope P from the space of its k-stresses together with the $(k-1)$-skeleton of P. The second concerns reconstructing the affine type of a d-polytope P, subject to certain additional conditions, from the space of its k-stresses. Strictly speaking, the conjecture only addresses the case $k = 2$, but it is very natural to extend it to higher values of k. Both conjectures only make sense when $d \geq 2k$.

With Satoshi Murai and Hailun Zheng[44], we proved the second conjecture whenever $d > 2k$, but the case $d = 2k$ remains wide open. With Hailun Zheng[49], we proved the first conjecture in the case $k = 2$, the first nontrivial case, but the cases $k > 2$ are again completely open at present. In fact, we proposed two stronger conjectures, which are also open for $k > 2$. So there is still much to do.

Mansour: In *Many neighborly spheres*, you and Hailun Zheng[50] show that there exist enormously many combinatorial types of neighborly spheres. How do you think about the contrast between extremality and abundance in this context? Does the existence of so many neighborly spheres change the way we should think about the geometry or topology underlying neighborliness?

Novik: Before discussing the number of neighborly spheres, let me first say a word about the total number of simplicial spheres. By constructions of Kalai[37], Pfeifle–Ziegler[51], and Nevo–Santos–Wilson[52], we know that for suf-

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[49] I. Novik and H. Zheng, *Reconstructing simplicial polytopes from their graphs and affine 2-stresses*, Israel J. Math. 255:2 (2023), 891–910.

[50] I. Novik and H. Zheng, *Many neighborly spheres*, Math. Ann. 388:1 (2024), 969–984.

[51] J. Pfeifle and G. M. Ziegler, *Many triangulated 3-spheres*, Math. Ann. 330:4 (2004), 829–837.

[52] E. Nevo, F. Santos, and S. Wilson, *Many triangulated odd-dimensional spheres*, Math. Ann. 364 (2016), no. 3–4, 737–762.

ficiently large n and $d \geq 4$, the total number $s(d,n)$ of $(d-1)$-spheres with n vertices is enormous: at least $2^{A_d n^{\lfloor d/2 \rfloor}}$ for some positive constant A_d. On the other hand, Stanley's Upper Bound Theorem[32] gives an upper bound of $2^{C_d n^{\lfloor d/2 \rfloor} \log n}$ for some constant $C_d > 0$.

With Hailun Zheng, we showed that, for $d \geq 5$, the number $\mathrm{sn}(d,n)$ of neighborly $(d-1)$-spheres with n vertices is also very large: at least $2^{B_d n^{\lfloor (d-1)/2 \rfloor}}$ for some constant $B_d > 0$. Of course for even d this is much smaller than the Nevo–Santos–Wilson bound. In other words, at present, we are very far from understanding what fraction of all simplicial spheres is neighborly. Gil[37] made a very bold conjecture that this fraction is asymptotically as large as it could possibly be: as $n \to \infty$, the ratio $\log(\mathrm{sn}(d,n))/\log(s(d,n))$ approaches 1.

I should also mention that all known constructions of "many" spheres and "many" neighborly spheres are still quite tame—they are either shellable or believed to be shellable. This suggests that we are still seeing only a small portion of the full picture, and the extremal objects likely form a much richer family than is currently understood.

Mansour: You have given talks at numerous conferences, workshops, and seminars. What role do you think such activities play in a researcher's development and in shaping the direction of a field?

Novik: I think that conferences, workshops, colloquia, and seminars play an extremely important role both in a researcher's development and in shaping a field. Preparing a talk forces you to decide which results and ideas to highlight and how best to explain them, and in doing so you often see these results in a new light. This process alone can generate new insights, and when combined with audience feedback and the questions that follow a talk, it can lead to unexpected collaborations, directions, and ideas.

In the same vein, I find that listening to talks is often much more efficient than reading papers. Papers record results and proofs, which can become quite technical, while talks emphasize the ideas and frequently convey the underlying insights: what the motivation was, what worked, what didn't, and how the ideas connect to other problems.

Finally, participating in a conference allows you to immerse yourself in a field for a few days without everyday distractions. That kind of focused environment can be incredibly energizing. In short, such activities are essential for a field to thrive.

Mansour: Would you tell us about your interests besides mathematics?

Novik: I enjoy simple things: spending time with my family, reading, walking, hiking, assembling jigsaw puzzles, and cooking. Doing these things usually helps me relax and reset.

Mansour: As an advisor, you have influenced the careers of many students. What advice do you have for young mathematicians who are just starting their academic journeys?

Novik: Work only on problems you enjoy. Do not be afraid of working on hard problems. One advantage of just starting out is that you don't yet know what other people have tried, so you can bring a fresh eye. This often turns out to be an advantage: it leads to trying and noticing new things that others have overlooked, which in turn can lead to very unexpected solutions. My graduate students often amaze me by solving long-standing problems I had thought were hopeless.

Mansour: Would you tell us about your thought process for the proof of one of your favorite results? How did you become interested in that problem? How long did it take you to figure out a proof? Did you have a "eureka moment"?

Novik: I would like to tell a story about one of my very first research results. The Upper Bound Theorem for simplicial spheres, proved by Richard Stanley[32] in 1975, asserts that among all $(d-1)$-spheres with n vertices, the boundary complex of the cyclic polytope—or, in fact, any $\lfloor d/2 \rfloor$-neighborly sphere—simultaneously maximizes all face numbers. Vic Klee[38], in 1964, conjectured that the same should hold for all Eulerian complexes of dimension $d-1$. In this generality, the conjecture is still open. Since every odd-dimensional simplicial manifold is Eulerian, a natural special case is to try to prove the conjecture for this class of complexes.

When I was just beginning my Master's studies, Gil Kalai sat me down and told me about this problem. He also summarized for me several key results in the area. First, instead of working with face numbers, one usually studies the h-numbers, obtained from the f-numbers by an invertible linear transformation. The h-numbers of a simplicial $(d-1)$-sphere—and, more generally, of any odd-dimensional manifold—form a symmetric sequence, that is, $h_i = h_{d-i}$ for all i; these are the Dehn–Sommerville relations.

Stanley's proof of the Upper Bound Theorem shows that for every $0 \leq i \leq d$, the h_i-number of a $(d-1)$-sphere with n vertices is the dimension of the i-th graded component of a certain quotient of a polynomial ring in $n-d$ variables. Hence $h_i \leq \binom{n-d+i-1}{i}$, which, for $i \leq d/2$, is exactly the h_i-number of the cyclic polytope. This does not hold for manifolds, but by a result of Schenzel[53] from 1981, the dimension of the corresponding graded component is given by the sum of h_i and a certain alternating weighted sum of the Betti numbers—topological invariants of the manifold. In other words, for $i \leq d/2$, we can still bound h_i from above by the h_i of the cyclic polytope plus an alternating weighted sum of Betti numbers.

Gil wanted me to start thinking about this problem. At that point, I barely knew what a simplicial complex was, but I did not want to disappoint him. So before our next meeting, I tried the only idea I could think of: the number of facets of any complex is the sum of its h-numbers. By the Dehn–Sommerville relations, this can be rewritten using only the first half of the h-numbers. Then one can upper bound each summand by the expression involving the h-numbers of the cyclic polytope plus the alternating Betti-number sum.

To my surprise, when I carried out the computation, I realized that in the resulting upper bound *each* Betti number appeared with a nonpositive coefficient. As a result, the entire sum was bounded above by the corresponding expression for the cyclic polytope—which is exactly the number of facets of the cyclic polytope. This was completely unexpected—and exhilarating.

Mansour: Is there a specific problem you have been working on for many years? What progress have you made?

Novik: I keep returning to several problems. One such problem was the g-theorem for spheres, but it has now been resolved. Another is Klee's upper bound conjecture for Eulerian complexes that are not manifolds. With Patricia Hersh and later with Ed Swartz, we made some progress, but so far we have only been able to prove the conjecture for pseudomanifolds with very isolated singularities.

A related problem—one that has gained even more importance after the proof of the g-conjecture—is to understand, for a pseudomanifold Δ that is not a manifold, the Hilbert function of the generic Artinian reduction of its Stanley–Reisner ring, as well as of a certain further quotient of this ring. Again with Ed Swartz, we obtained an answer for pseudomanifolds with very isolated singularities, but we have not yet been able to move beyond this case.

With Hailun Zheng, we have also been trying to better understand the top space of affine stresses for spheres with additional properties—that is, the space of affine k-stresses of $(2k-1)$-spheres. We make occasional progress, but a full picture is still out of reach.

Mansour: In a very recent short article[54], published in the Newsletter of the European Mathematical Society, Professor Melvyn B. Nathanson, while elaborating on the ethical aspects of the question "Who Owns the Theorem?", concluded that "Mathematical truths exist and mathematicians only discover them." On the other side, there are opinions that "mathematical truths are invented". As a third way, some people claim that it is both invented and discovered. What do you think about this old discussion? More precisely, do you believe that you *invent* or *discover* your theorems?

Novik: I believe that we discover our theorems. More strongly, I believe that the theorems "find" us.

Mansour: Professor Novik, I would like to thank you for this very interesting interview on behalf of the journal Enumerative Combinatorics and Applications.

Novik: Thank you. It was an honor.

[53] P. Schenzel, *On the number of faces of simplicial complexes and the purity of Frobenius*, Math. Z. 178 (1981), 125–142.

[54] M. B. Nathanson, *Who Owns the Theorem?* The best writing on Mathematics 2021, Princeton: Princeton University Press, 2022, 255–257.